

# KSETA topical courses

## Neutrino physics III: Neutrino Mass

Thomas Schwetz-Mangold



Karlsruhe, 7-8 Oct 2020

## Neutrinos oscillate...



... and have mass  $\Rightarrow$  physics beyond the Standard Model

- ▶ Lecture I: Neutrino Oscillations
- ▶ Lecture II: Neutrinos in Cosmology
- ▶ Lecture III: Neutrino mass - Dirac versus Majorana
- ▶ Lecture IV: Neutrinos and physics beyond the Standard Model

# Neutrinos oscillate...



... and have mass  $\Rightarrow$  physics beyond the Standard Model

- ▶ Lecture I: Neutrino Oscillations
- ▶ Lecture II: Neutrinos in Cosmology
- ▶ **Lecture III: Neutrino mass - Dirac versus Majorana**
- ▶ Lecture IV: Neutrinos and physics beyond the Standard Model

# Outline

## Absolute neutrino mass

- Beta decay – the KATRIN experiment
- Neutrinoless double-beta decay

## Fermion masses

- Dirac mass
- Majorana mass
- Dirac versus Majorana neutrinos in the SM

## The Standard Model and neutrino mass

# Outline

## Absolute neutrino mass

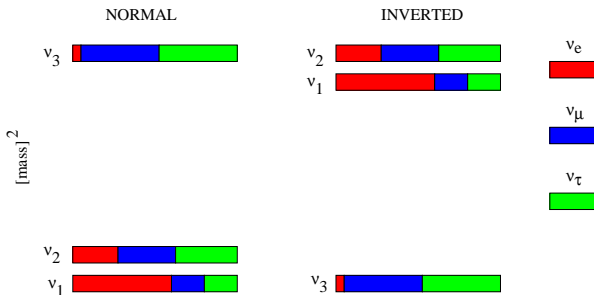
- Beta decay – the KATRIN experiment
- Neutrinoless double-beta decay

## Fermion masses

- Dirac mass
- Majorana mass
- Dirac versus Majorana neutrinos in the SM

## The Standard Model and neutrino mass

- ▶ neutrino oscillations only determine  $\Delta m_{ij}^2$
- ▶ absolute mass scale is not constrained



# Absolute neutrino mass

Three ways to measure absolute neutrino mass:

- ▶ Cosmology  
(with caveats: cosmological model/data selection)
- ▶ Endpoint of beta spectrum:  ${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$   
(experimentally challenging  $\rightarrow$  KATRIN)
- ▶ Neutrinoless double beta-decay:  $(A, Z) \rightarrow (A, Z + 2) + 2e^-$   
(with caveats: lepton number violation)

# Absolute neutrino mass

Three ways to measure absolute neutrino mass:  
sensitive to different quantities

- ▶ Cosmology

(with caveats: cosmological model/data selection)

$$\sum_i m_i$$

- ▶ Endpoint of beta spectrum:  ${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$

(experimentally challenging  $\rightarrow$  KATRIN)

$$m_\beta^2 = \sum_i |U_{ei}^2| m_i^2$$

- ▶ Neutrinoless double beta-decay:  $(A, Z) \rightarrow (A, Z + 2) + 2e^-$

(with caveats: lepton number violation)

$$m_{ee} = \left| \sum_i U_{ei}^2 m_i \right|$$



# Beta decay

$$N(A, Z) \rightarrow N(A, Z + 1) + e^- + \bar{\nu}_e$$

$$\frac{d\Gamma}{dE_e} = \frac{G_F^2 m_e^5}{2\pi^2} \cos^2 \theta_c |\mathcal{M}|^2 F(Z, E_e) \underbrace{E_e p_e E_\nu p_\nu}_{\text{phase space}}$$

Tritium decay:  ${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$

$$M_{{}^3\text{H}} = 2.808\,920\,8205 \times 10^6 \text{ keV}$$

$$M_{{}^3\text{He}} = 2.808\,391\,2193 \times 10^6 \text{ keV}$$

$$m_e = 510.9989 \text{ keV}$$

$$Q \equiv M_{{}^3\text{H}} - M_{{}^3\text{He}} - m_e = 18.6023 \text{ keV} \ll M_{{}^3\text{H}}, M_{{}^3\text{He}}$$

$$\kappa \equiv M_{{}^3\text{He}}/M_{{}^3\text{H}} = 1 - 1.89 \times 10^{-4}$$

# Beta decay

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# Tritium beta decay

use E-momentum conservation, calculate electron kin. energy:

$$T \equiv E_e - m_e = \frac{1}{2M_{3\text{H}}} \left[ (M_{3\text{H}} - m_e)^2 - M_{3\text{He}}^2 - 2M_{3\text{He}}E_\nu \right]$$

$T$  has a maximum when  $E_\nu$  has a minimum:

$$\begin{aligned} m_\nu = 0: \quad T_{\max,0} &= \frac{1}{2M_{3\text{H}}} \left[ (M_{3\text{H}} - m_e)^2 - M_{3\text{He}}^2 \right] \\ &= Q - \frac{(M_{3\text{H}} - M_{3\text{He}})^2}{2M_{3\text{H}}} \approx Q - 3.4 \text{ eV} \end{aligned}$$

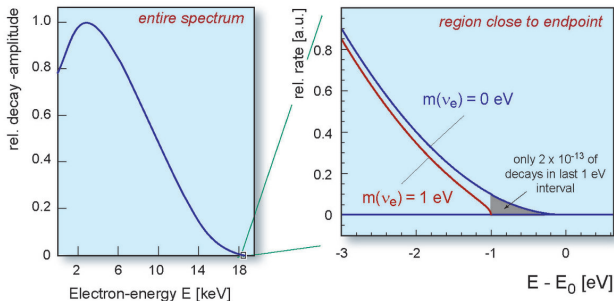
$$m_\nu > 0: \quad T_{\max} = T_{\max,0} - \kappa m_\nu$$

$\Rightarrow$  finite neutrino mass leads to a shift in electron spectrum endpoint

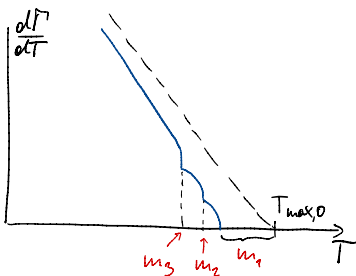
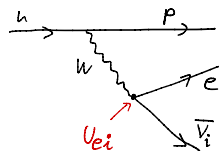
# Tritium decay spectrum close to the endpoint

phase space factor:  $E_\nu p_\nu = E_\nu \sqrt{E_\nu^2 - m_\nu^2}$ , use  $E_\nu \approx \frac{M_{3\text{H}}}{M_{3\text{He}}}(T_{\text{max},0} - T)$ :

$$\frac{d\Gamma}{dT} \propto (T_{\text{max},0} - T) \sqrt{(T_{\text{max},0} - T)^2 - \kappa^2 m_\nu^2}$$



# Take into account neutrino mixing



incoherent sum of individual mass states:

$$\frac{d\Gamma}{dT} = \sum_i |U_{ei}|^2 \frac{d\Gamma_i}{dT}$$

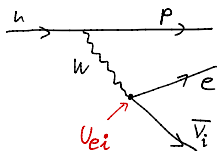
$$\propto (T_{max,0} - T) \sum_i |U_{ei}|^2 \sqrt{(T_{max,0} - T)^2 - \kappa^2 m_i^2}$$

for  $T_{max,0} - T \gg \Delta m$ :

$$\frac{d\Gamma}{dT} \approx (T_{max,0} - T) \sqrt{(T_{max,0} - T)^2 - \kappa^2 m_\beta^2}$$

$$m_\beta^2 \equiv \sum_i |U_{ei}|^2 m_i^2$$

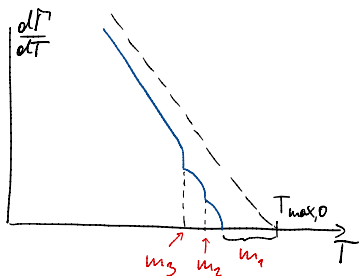
# Take into account neutrino mixing



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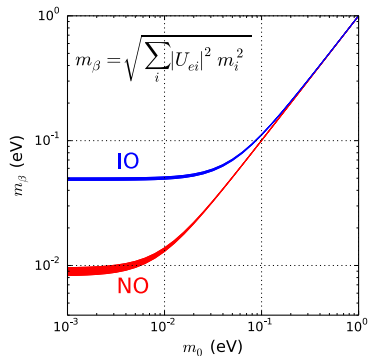
# The effective mass

$$m_\beta^2 \equiv \sum_i |U_{ei}|^2 m_i^2 \approx \begin{cases} m_0^2 + |U_{e2}|^2 \Delta m_{21}^2 + |U_{e3}|^2 \Delta m_{31}^2 & \text{(NO)} \\ m_0^2 + (1 - |U_{e3}|^2) |\Delta m_{31}^2| & \text{(IO)} \end{cases}$$

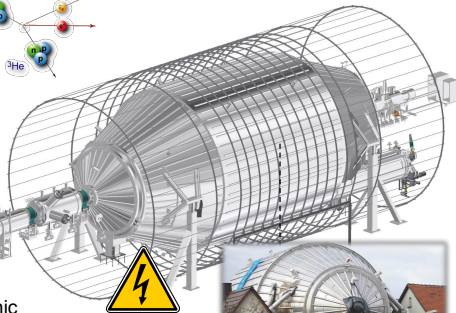
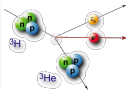
minimum values for  $m_0 = 0$ :

$$m_\beta^{\min} \approx \begin{cases} 9 \text{ meV} & \text{(NO)} \\ 50 \text{ meV} & \text{(IO)} \end{cases}$$

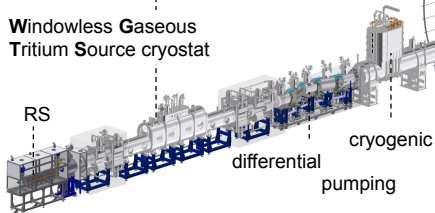
for  $m_0 \gg |\Delta m_{31}^2|$ :  $m_\beta \approx m_0$



# KATRIN overview: 70 m long beamline



Windowless Gaseous  
Tritium Source cryostat

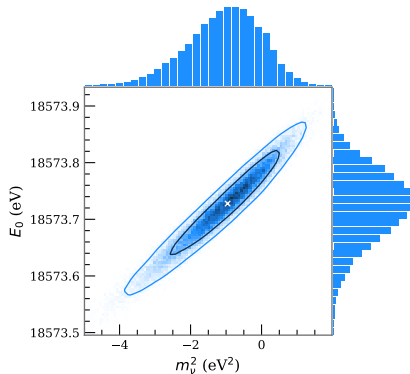
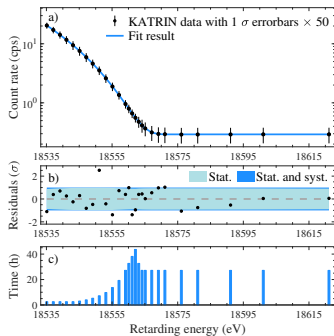


Main Spectrometer



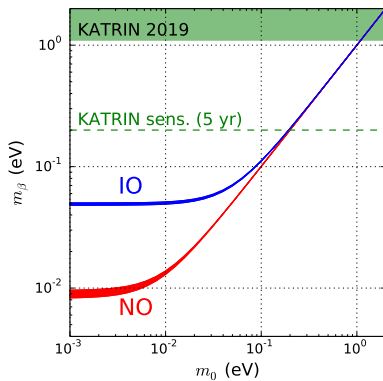
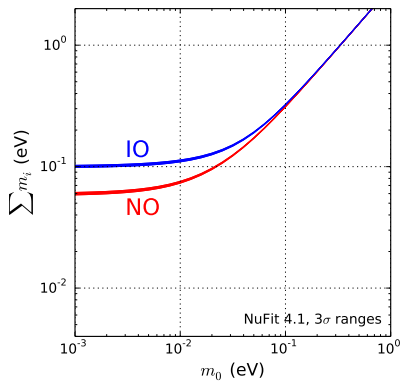


## KATRIN 2019 Aker et al., 1909.06048

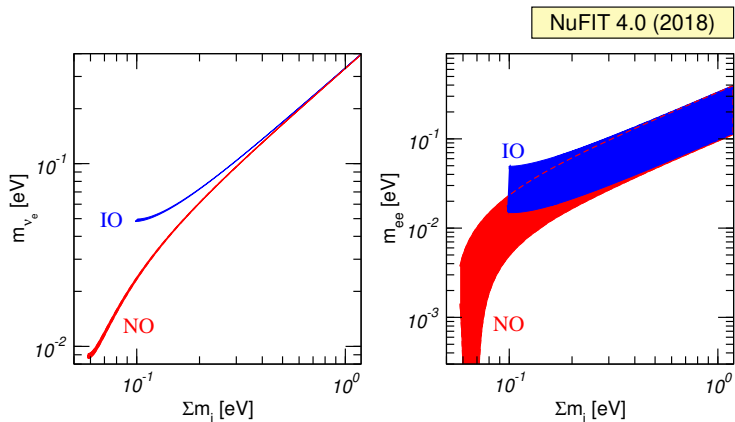


$$m_\beta^2 = -1.0_{-1.1}^{+0.9} \text{ eV}^2$$

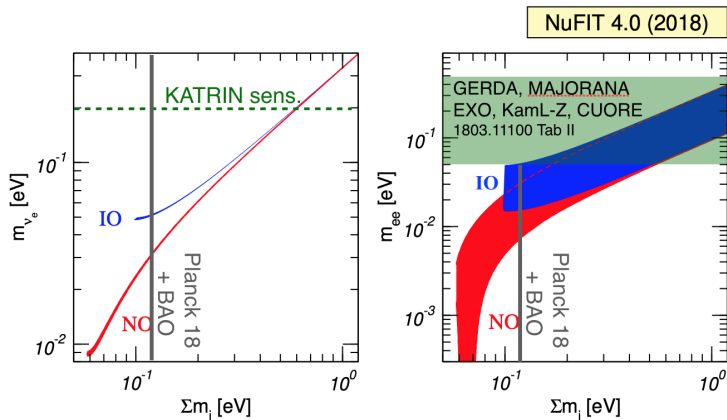
$$m_\beta < 1.1 \text{ eV (90\% CL)}$$

Cosmology and  $\beta$  decay observables

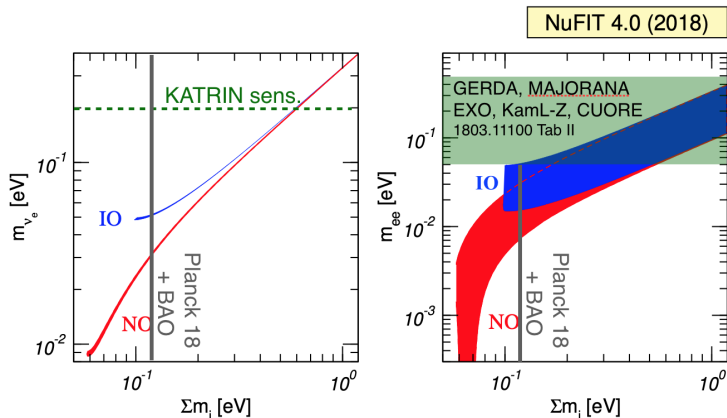
# Absolute neutrino mass



# Absolute neutrino mass



# Absolute neutrino mass



relies on standard three-flavour scenario and standard cosmology

Any inconsistency would indicate new physics beyond 3 flavour neutrino mass!

# Relaxing the neutrino mass bound with neutrino decay

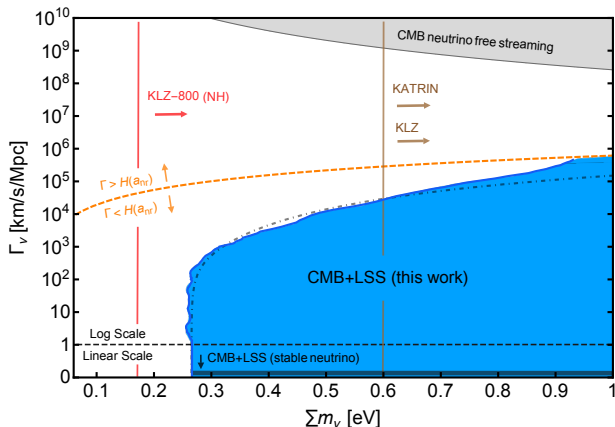
Chacko, Dev, Du, Poulin, Tsai,  
1909.05275, 2002.08401

assume neutrino decay:

$$\nu_i \rightarrow \nu_4 \phi$$

with  $i = 1, 2, 3$  and  
 $m_4, m_\phi = 0$

decay rate:  $\Gamma_\nu$



see also Escudero, Fairbairn, 1907.05425; Escudero, Lopez-Pavon, Rius, Sandner, 2007.04994

# Neutrinoless double-beta decay

2-neutrino double-beta decay:  $(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}_e$

- ▶ 2nd order in  $G_F$
- ▶ only observable if single beta decay forbidden
- ▶ 35 natural isotopes are known (all even-even nuclei)
- ▶  $T_{1/2}^{2\nu} \sim 10^{18} - 10^{20}$  yr ( $\gg t_{\text{Universe}}$ )

neutrinoless double-beta decay:  $(A, Z) \rightarrow (A, Z + 2) + 2e^-$

- ▶ no neutrinos emitted
- ▶ violation of lepton number by two units
- ▶ sum of electron kinetic energies  $T = T_1 + T_2 = Q_{2\beta} = M_i - M_f - 2m_e$
- ▶ Examples:  ${}^{76}\text{Ge} \rightarrow {}^{76}\text{Se}$      $Q_{2\beta} = 2.039$  MeV  
 ${}^{136}\text{Xe} \rightarrow {}^{136}\text{Ba}$      $Q_{2\beta} = 2.468$  MeV

# Neutrinoless double-beta decay

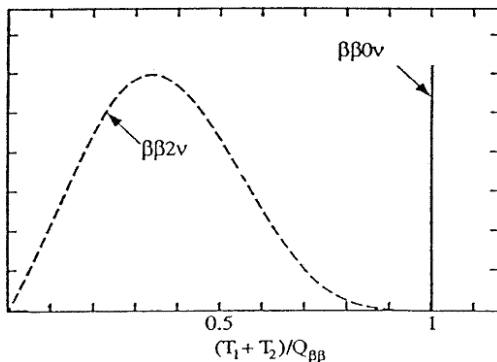
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Example  $^{76}\text{Ge}$  (GERDA experiment):

$$2\beta 2\nu : T_{1/2} = (1.8 \pm 0.1) \times 10^{21} \text{ yr}$$

$$2\beta 0\nu : T_{1/2} > 2.1 \times 10^{25} \text{ yr}$$

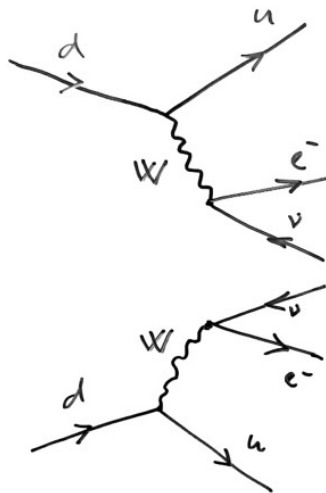
(importance of energy resolution and background suppression)

# Neutrinoless double-beta decay



- ▶ an observation of this process would prove that **lepton number is violated**
- ▶ proves Majorana nature of neutrinos
- ▶ BUT no direct prove of neutrino mass  
(a different mechanism could be responsible)

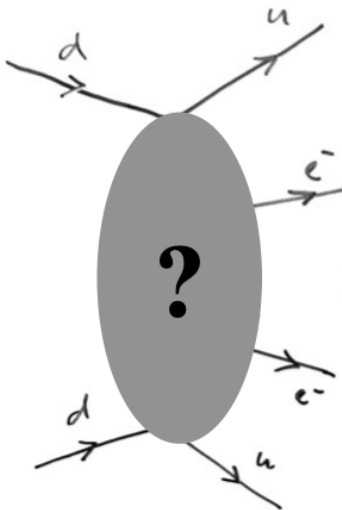
# The neutrino-mass mechanism



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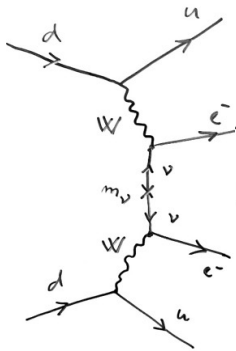


BUT: what we observe is just  $\Delta L = 2$

# The neutrino-mass mechanism

assuming that light neutrino exchange is responsible for the decay:

$$\begin{aligned}
 m_{\beta\beta} &= |\mathcal{M}_{ee}| \quad (\text{in basis where ch. lepton mass matrix is diag.}) \\
 &= \left| \sum_i U_{ei}^2 m_i \right| = |c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_1} m_2 + s_{13}^2 e^{i\alpha_2} m_3|
 \end{aligned}$$



coherent sum of individual neutrino masses

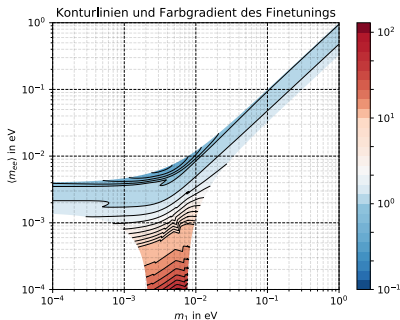
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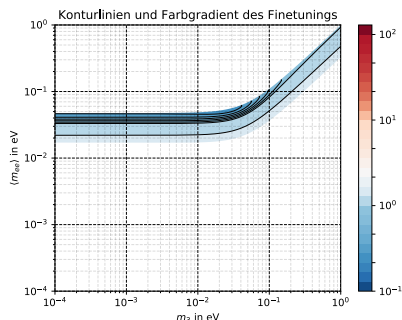
$$m_{\beta\beta} = |\mathcal{M}_{ee}| \quad (\text{in basis where ch. lepton mass matrix is diag.})$$

$$= \left| \sum_i U_{ei}^2 m_i \right| = |c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_1} m_2 + s_{13}^2 e^{i\alpha_2} m_3|$$

normal ordering

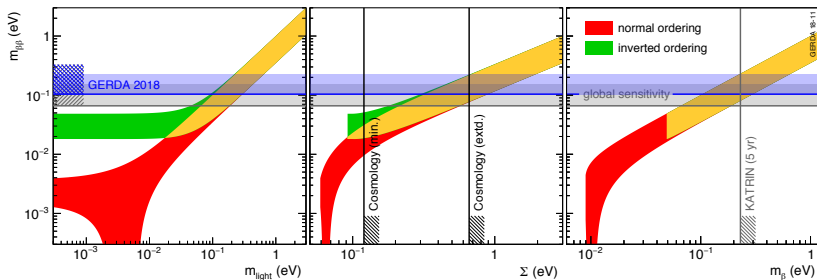


inverted ordering



M. Eichhorn, BSc thesis, KIT 2018

| Experiment        | Isotope           | $M_i$<br>(kmol) | FWHM<br>(keV) | $\mathcal{L}(T_{1/2})$<br>( $10^{25}$ yr) | $\mathcal{S}(T_{1/2})$<br>( $10^{25}$ yr) | $m_{\beta\beta}$<br>(meV) |
|-------------------|-------------------|-----------------|---------------|-------------------------------------------|-------------------------------------------|---------------------------|
| GERDA (this work) | $^{76}\text{Ge}$  | 0.41            | 3.3           | 9                                         | 11                                        | 104 - 228                 |
| Majorana [22]     | $^{76}\text{Ge}$  | 0.34            | 2.5           | 2.7                                       | 4.8                                       | 157 - 346                 |
| CUPID-0 [23]      | $^{82}\text{Se}$  | 0.063           | 23            | 0.24                                      | 0.23                                      | 394 - 810                 |
| CUORE [24]        | $^{130}\text{Te}$ | 1.59            | 7.4           | 1.5                                       | 0.7                                       | 162 - 757                 |
| EXO-200 [25]      | $^{136}\text{Xe}$ | 1.04            | 71            | 1.8                                       | 3.7                                       | 93 - 287                  |
| KamLAND-Zen [26]  | $^{136}\text{Xe}$ | 2.52            | 270           | 10.7                                      | 5.6                                       | 76 - 234                  |
| Combined          |                   |                 |               |                                           |                                           | 66 - 155                  |



GERDA Coll., 1909.02726



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# Dirac fermion

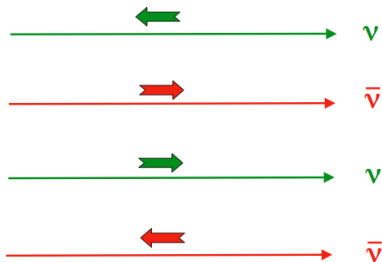
$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

Dirac equation:

$$(i\gamma^\mu\partial_\mu - m)\psi = 0$$

$\psi$  is a 4-component object: 2 helicity states for particle and anti-particle

4 mass-degenerate states:



# Chirality

parity is violated in weak interactions

left and right chiral fields transform differently under SM gauge group

left- and right-chirality projection operators:

$$P_L = \frac{1}{2}(1 - \gamma_5), \quad P_R = \frac{1}{2}(1 + \gamma_5)$$

left and right chiral fields (irreducible representations of Lorentz group):

$$P_L \psi_L = \psi_L, \quad P_R \psi_R = \psi_R, \quad \psi = \psi_L + \psi_R$$

Dirac Lagrangian:

$$\begin{aligned}\mathcal{L}_D &= i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi \\ &= i\bar{\psi}_L\gamma^\mu\partial_\mu\psi_L + i\bar{\psi}_R\gamma^\mu\partial_\mu\psi_R - m\bar{\psi}_L\psi_R - m\bar{\psi}_R\psi_L\end{aligned}$$

Dirac equation:

$$\begin{aligned}i\gamma^\mu\partial_\mu\psi_L - m\psi_R &= 0 \\ i\gamma^\mu\partial_\mu\psi_R - m\psi_L &= 0\end{aligned}$$

mass term mixes chiralities

for mass-less fermion the equations of motions for left- and right-chiral fields decouple  $\rightarrow$  Weyl equation:

$$\begin{aligned}i\gamma^\mu\partial_\mu\psi_L &= 0 \\ i\gamma^\mu\partial_\mu\psi_R &= 0\end{aligned}$$

Dirac Lagrangian:

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invariant under a  $U(1)$  symmetry

$$\psi_L \rightarrow e^{i\alpha}\psi_L, \quad \psi_R \rightarrow e^{i\alpha}\psi_R$$

conserved quantum number (charge, lepton number, ...)

particle is different from anti-particle

$\Rightarrow$  any charged Fermion has to be a Dirac particle

define particle- antiparticle conjugation  $\hat{C}$ :

$$\hat{C}: \quad \psi \rightarrow \psi^c \equiv C\bar{\psi}^T \equiv C\gamma_0^T\psi^*$$

with

$$C^{-1}\gamma^\mu C = -\gamma^{\mu T}, \quad C^\dagger = C^{-1} = -C^*$$

note that  $\hat{C}$  changes chirality:

$$\psi_L \rightarrow (\psi_L)^c \equiv \psi_L^c \quad \text{with} \quad P_R\psi_L^c = \psi_L^c, \quad P_L\psi_L^c = 0$$

Majorana field: replace  $\psi_R$  by  $\psi_L^c$ :

$$\psi = \psi_L + \psi_L^c$$

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**Majorana field:** replace  $\psi_R$  by  $\psi_L^c$ :

$$\psi = \psi_L + \psi_L^c$$

# Majorana fermion

the Majorana field  $\psi = \psi_L + \psi_L^c$  fulfills the Majorana condition

$$\psi = \psi^c$$

“is its own anti-particle”

only 2 independent  
(mass-degenerate) states:



# Majorana fermion

Lagrangian for a Majorana fermion

$$\mathcal{L}_M = i\bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + \mathcal{L}_{\text{mass}}$$

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= -\frac{m}{2} [\bar{\psi}_L^c \psi_L + \bar{\psi}_L \psi_L^c] \\ &= +\frac{m}{2} [\psi_L^T C^{-1} \psi_L - \bar{\psi}_L C \bar{\psi}_L^T] = \frac{m}{2} [\psi_L^T C^{-1} \psi_L + \text{h.c.}] \end{aligned}$$

explicitly built out of only  $\psi_L$  (2 dof)

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using  $\psi = \psi_L + \psi_L^c$  and dropping a term with a total derivative:

$$\mathcal{L}_M = \frac{i}{2} \bar{\psi} \gamma^\mu \partial_\mu \psi - \frac{m}{2} \bar{\psi} \psi$$

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Majorana equation:

$$i\gamma^\mu \partial_\mu \psi_L - m\psi_L^c = 0$$

this form holds for representations of the  $\gamma$ -matrices where  $C$  is real, i.e., where  $C^{-1} = -C$ , e.g.,  $C = i\gamma^2\gamma^0$

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$$\mathcal{L}_M = i\bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + \frac{m}{2} \left[ \psi_L^T C^{-1} \psi_L + \text{h.c.} \right]$$

this Lagrangian is not invariant under  $\psi_L \rightarrow e^{i\alpha} \psi_L$

Majorana mass term breaks all  $U(1)$  charges by 2 units

cannot define “particle” and “anti-particle”

any (electrically) charged particle cannot be a Majorana particle

In weak interactions we speak about  
“neutrinos” and “antineutrinos”

How can the neutrino be a Majorana particle,  
being its own antiparticle?

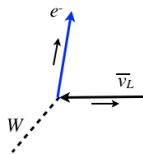
In the SM neutrinos are massless and only left-chiral fields participate in weak interactions:

$$\begin{aligned}\mathcal{L}_{\text{CC}} &= -\frac{g}{\sqrt{2}} W^\rho \bar{\ell}_L \gamma_\rho \nu_L + \text{h.c.} \\ &= -\frac{g}{\sqrt{2}} W^\rho \bar{\ell}_L \gamma_\rho \nu_L - \frac{g}{\sqrt{2}} W^{\rho\dagger} \bar{\nu}_L \gamma_\rho \ell_L\end{aligned}$$

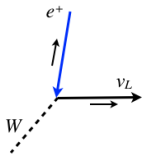
- ▶ the field  $\nu_L$  contains two helicity states
- ▶ for massless fermions helicity states correspond to chiral states
- ▶ the left-handed field  $\nu_L$  acts as “neutrino”  
the right-handed field  $\bar{\nu}_L$  acts as “antineutrino”



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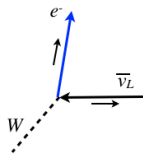


outgoing  
“antineutrino”  
(right-handed field  $\bar{\nu}_L$ )  
produced together with  
negative charged lepton

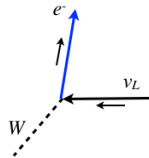


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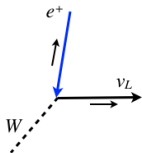
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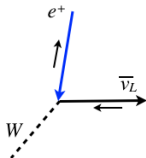
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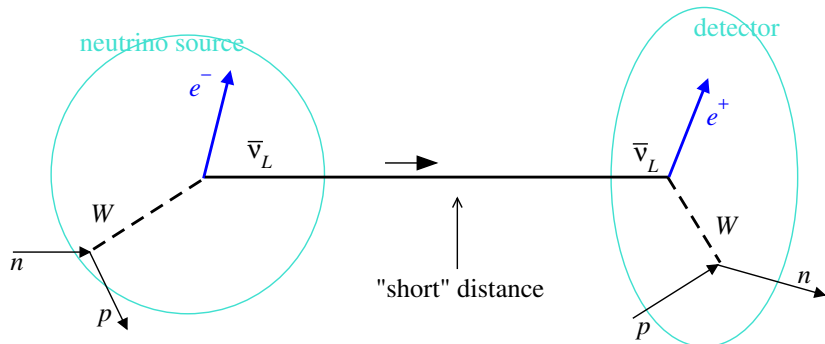


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# A typical neutrino experiment



- ▶ we need a L and a R neutrino state for weak interactions (to describe “neutrino” and “antineutrino”)
- ▶ we need a L and a R neutrino state to form a mass term

### Majorana:

- ▶ those states are identical (there are only two independent states)

### Dirac:

- ▶ the R state to form the mass term is different than the one acting as “antineutrino” in weak interactions (4 independent states) → “right-handed neutrino”: does not participate in weak interactions

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## Chirality versus helicity

physical states are helicity eigenstates:

$$\frac{\vec{\sigma}\vec{p}}{|\vec{p}|}\psi_{\pm} = \pm\psi_{\pm}$$

for massless fermions helicity and chirality coincides:

$$\psi_{-} = \psi_{L}, \quad \psi_{+} = \psi_{R} \quad (\text{massless})$$

for relativistic massive fermions ( $m \ll E$ ) we have:

$$\psi_{-} \approx \psi_{L} + \frac{m}{2E}\psi_{R}, \quad \psi_{+} \approx \psi_{R} + \frac{m}{2E}\psi_{L}$$

OBS: here “ $\psi_{R}$ ” denotes the right-chiral field in the mass term, which corresponds to  $\psi^{c}$  in the Majorana case

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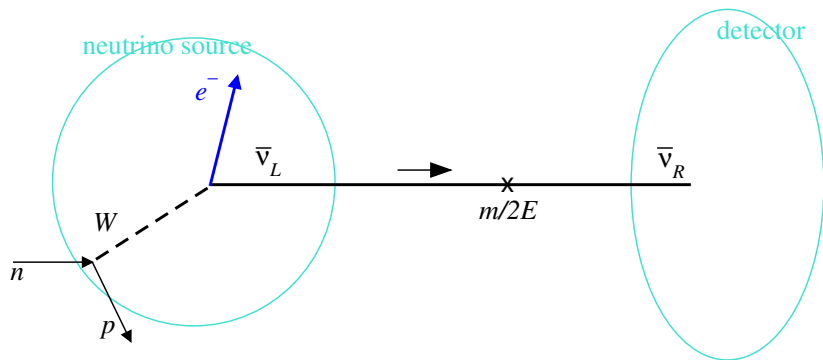
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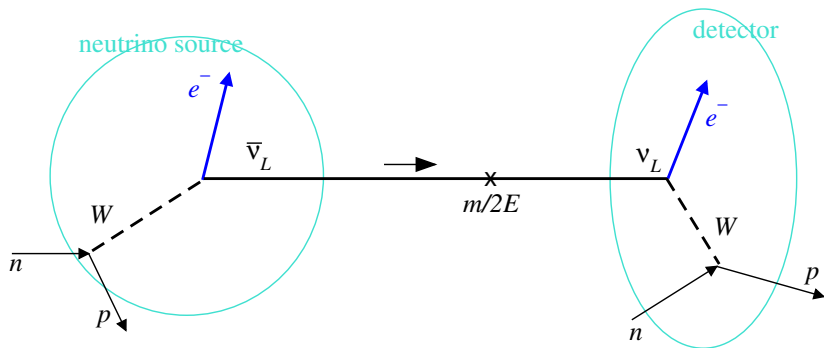
# Mass induced chirality flip - Dirac

with a probability suppressed wrt leading diagram by  $(m/2E)^2 \lesssim 10^{-12}$



# Mass induced chirality flip - Majorana

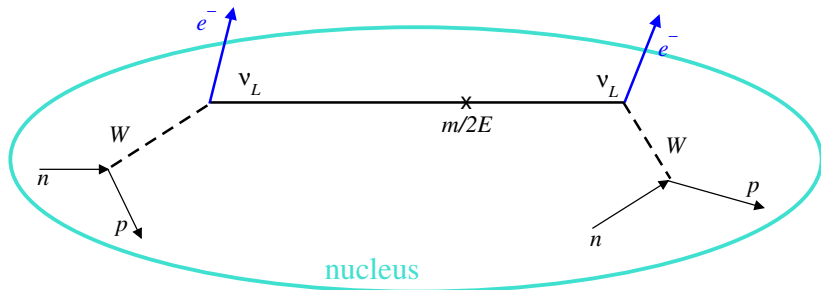
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Schechter, Valle, PRD 1981

# Mass induced chirality flip - Majorana

Neutrinoless double-beta decay  $(A, Z) \rightarrow (A, Z + 2) + 2e^-$



# Outline

## Absolute neutrino mass

- Beta decay – the KATRIN experiment
- Neutrinoless double-beta decay

## Fermion masses

- Dirac mass
- Majorana mass
- Dirac versus Majorana neutrinos in the SM

## The Standard Model and neutrino mass

# Masses in the Standard Model

- ▶ The Standard Model has only one dimension full parameter: the vacuum expectation value of the Higgs:

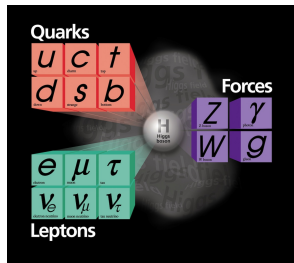
$$\langle \phi \rangle \approx 174 \text{ GeV}$$

- ▶ All masses in the Standard Model are set by this single scale:

$$m_i = y_i \langle \phi \rangle$$

top quark:  $y_t \approx 1$

electron:  $y_e \approx 10^{-6}$



# Fermion masses in the Standard Model

fermions of one generation:

$$\text{quarks: } Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, u_R, d_R \quad \text{leptons: } L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, e_R$$

mass terms from Yukawa coupling to Higgs  $\phi$

$$\mathcal{L}_Y = -\lambda_d \bar{Q}_L \phi d_R - \lambda_u \bar{Q}_L \tilde{\phi} u_R + \text{h.c.} \quad -\lambda_e \bar{L}_L \phi e_R + \text{h.c.}$$

$$\text{EWSB} \rightarrow -m_d \bar{d}_L d_R - m_u \bar{u}_L u_R + \text{h.c.} \quad -m_e \bar{e}_L e_R + \text{h.c.}$$

$$\tilde{\phi} \equiv i\sigma_2 \phi^*, \quad m_d = \lambda_d \frac{v}{\sqrt{2}}, \quad m_u = \lambda_u \frac{v}{\sqrt{2}}, \quad m_e = \lambda_e \frac{v}{\sqrt{2}}, \quad \langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Dirac mass terms for charged fermions

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**Dirac mass terms for charged fermions**



- ▶ “right-handed” neutrinos would be complete gauge singlets in the SM
- ▶ no gauge interactions
- ▶ left out in the original formulation of the SM  
⇒ no Dirac mass term for neutrinos
  
- ▶ Why is there no Majorana mass term?
  
- ▶ Lepton-number is an accidental symmetry in the SM → given the gauge symmetry and the field content of the SM we cannot construct a Majorana mass term for neutrinos (true at any loop order)

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## In the SM neutrinos are massless because. . .

1. there are no right-handed neutrinos to form a Dirac mass term
2. because of the field content (scalar sector) and gauge symmetry lepton number<sup>1</sup> is an accidental global symmetry of the SM and therefore no Majorana mass term can be induced.
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### **Neutrino mass implies physics beyond the Standard Model**

At least one of the above items needs to be violated

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