KSETA topical courses

Neutrino physics II: Neutrinos in Cosmology

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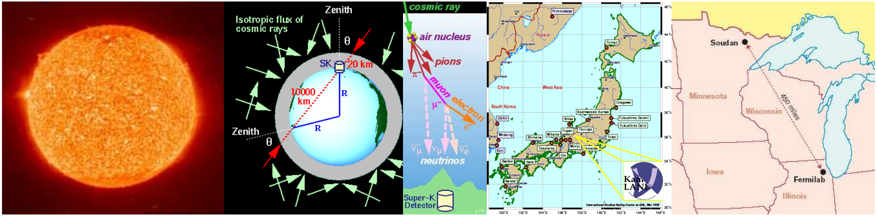
HIDDeν
Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

Karlsruhe, 7-8 Oct 2020

Neutrinos oscillate...



... and have mass ⇒ physics beyond the Standard Model

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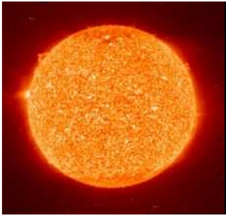

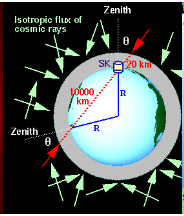

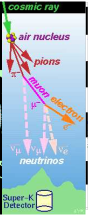

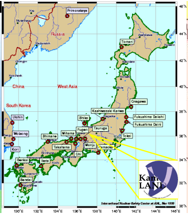

... and have mass ⇒ physics beyond the Standard Model

- Lecture I: Neutrino Oscillations
- Lecture II: Neutrinos in Cosmology
- Lecture III: Neutrino mass - Dirac versus Majorana
- Lecture IV: Neutrinos and physics beyond the Standard Model

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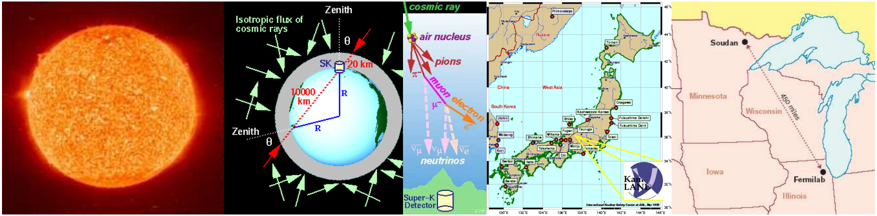

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Outline - Neutrino Physics II

ΛCDM cosmology
- Thermodynamics in the early Universe
- Cosmic neutrinos

Big Bang nucleosynthesis
- Counting neutrino flavours

Structure formation
- Effect of neutrinos on structure formation
- Neutrino mass bound from cosmology

Summary

Outline

Big Bang cosmology

the cosmological principle: universe is homogeneous and isotropic

+ general relativity
+ standard model of particle physics

observational pilars:

- Hubble diagram shows expansion
- Big Bang Nucleosynthesis (BBN)
- Cosmic microwave background (CMB)
- Distribution of structure at the largest scales

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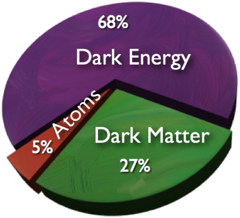
ΛCDM cosmology



DE:	$\Omega_\Lambda \approx 0.7$
CDM:	$\Omega_c \approx 0.26$
baryons:	$\Omega_b \approx 0.04$
radiation:	$\Omega_R \approx 10^{-5}$

Cosmic expansion

space-time configuration consistent with cosmological principle (homogeneous and isotropic): Friedman-Lemaitre-Robertson-Walker

$$ds^2 = dt^2 - a(t)^2(dV)^2$$

- 3-dim space dV can have positive, negative or zero curvature observations: very close to flat (predicted by Inflation) $(dV)^2 \to dx^2 + dy^2 + dz^2$
- $a(t)$... cosmic scale factor
- Hubble parameter $H(t) = \dot{a}(t)/a(t)$
- Hubble constant $H_0 = H(t_0)$, where t_0 denotes "today"

$$H_0 = 100\, h\, \mathrm{km/s/Mpc}\,, \qquad h \approx 0.7$$

$$1\,\mathrm{Mpc} = 10^6\,\mathrm{pc}\,, \quad 1\,\mathrm{pc} \approx 3.08 \times 10^{16}\,\mathrm{m} \approx 3.26\,\mathrm{ly}$$

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Energy density in the expanding Universe

- energy-momentum conservation in the expanding Universe:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(p + \rho) = 0$$

- energy density $\rho = E/V$, pressure p, equation of state: $p = w\rho$

cold matter (non-rel. particles)	$E = N\, mc^2$	$w = 0$	$\Rightarrow \rho \propto a^{-3}$
radiation (relativistic particles)	$E = N\, \hbar\omega = N\, \frac{\hbar}{2\pi\lambda}$	$w = 1/3$	$\Rightarrow \rho \propto a^{-4}$
cosmological constant Λ		$w = -1$	$\Rightarrow \rho = const$

$$\Rightarrow \quad \rho_{tot} = \rho_R(t_0)\left(\frac{a_0}{a}\right)^4 + \rho_M(t_0)\left(\frac{a_0}{a}\right)^3 + \rho_\Lambda$$

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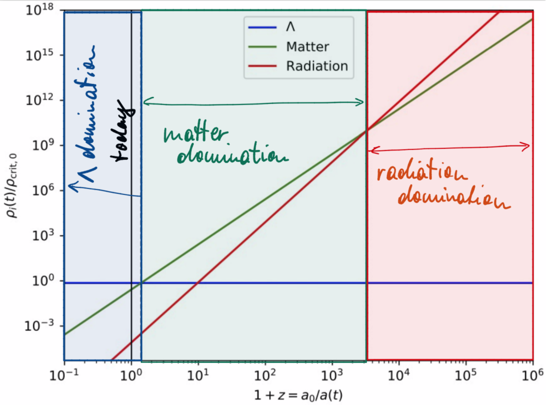

Dynamics of expansion $\rightarrow$ Friedman equation

Einstein equations + FLRW metric:

$$H^2(t) = \frac{\dot{a}^2}{a^2} = \frac{8\pi G_N}{3}\rho_{\rm tot}$$

$\rightarrow$ total energy density controls expansion rate of Universe

dynamics for $a(t)$ follow from Einstein equations:

R: $a(t) \propto \sqrt{t}$, M: $a(t) \propto t^{2/3}$, Λ: $a(t) \propto \exp(H_0\sqrt{\Omega_\Lambda}t)$

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phase space distribution function for a relativistic particle with $\mu = 0$:

$$f_{BE/FD}(p) = \frac{1}{e^{E/T} \mp 1}, \qquad p = |\vec{p}| \approx E$$

[indep. of $\vec{x}$ and direction of $\vec{p}$ due to cosmological principle]

number density:

$$n = g \int \frac{d^3p}{(2\pi)^3} f_{BE/FD}(p), \qquad n_B = g\frac{\zeta(3)}{\pi^2} T^3, \quad n_F = \frac{3}{4} n_B$$

energy density:

$$\rho = g \int \frac{d^3p}{(2\pi)^3} E\, f_{BE/FD}(p), \qquad \rho_B = g\frac{\pi^2}{30} T^4, \quad n_F = \frac{7}{8} n_B$$

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Expansion rate during radiation domination

energy density:

$$\rho = g \int \frac{d^3p}{(2\pi)^3} E\, f_{BE/FD}(p)\,, \qquad \rho_B = g\frac{\pi^2}{30} T^4\,, \quad n_F = \frac{7}{8} n_B$$

expansion rate during RD:

$$H = \sqrt{\frac{8\pi G_N}{3}\sum_{i\in R}\rho_i} \simeq \sqrt{g_{eff}}\frac{T^2}{M_{Pl}}\,, \qquad G_N = \frac{1}{M_{Pl}^2}$$

g_{eff} counts effective degrees of freedom of all relativistic particles

When is a particle specie in thermal equilibrium?

$$\boxed{\text{interactions} \Leftrightarrow \text{expansion}}$$

interaction rate: $\Gamma = n\langle v\sigma\rangle$

$\Gamma > H$: in thermal equilibrium
$\Gamma < H$: out of equilibrium

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Consider decoupled relativistic species

$$f_{BE/FD}(p)d^3p \propto \frac{E^2 dE}{e^{E/T} \mp 1}, \qquad p = |\vec{p}| \approx E$$

- redshift: $E = \hbar\omega \propto a^{-1}$
- thermal distribution is maintained for $T \propto a^{-1}$ after decoupling
- number density scales with $a^{-3} \propto T^3$,
  energy density with $a^{-4} \propto T^4$
  (same as for specie in thermal equilibrium)

Cosmic microwave background photons

- photons decouple around $t \sim 300\,000$ yr when Universe becomes neutral
- black body spectrum with $T_0 = 2.726 \pm 0.001\,K$
- number density of photons today: $n_\gamma = 2\frac{\zeta(3)}{\pi^2} T_0^3 \approx 410\,cm^{-3}$
- energy density of photons today:

$$\rho_\gamma = 2\frac{\pi^2}{30} T_0^4$$

$$\Omega_\gamma \equiv \frac{\rho_\gamma}{\rho_{crit}} \approx 2 \times 10^{-5} h^{-2}\,, \qquad \rho_{crit} = \frac{3H_0}{8\pi G_N}$$

Neutrino decoupling

- neutrino interactions with cosmic plasma: e.g.: $e^+e^- \leftrightarrow \nu\bar{\nu}$
- weak interactions: $\sigma \sim G_F^2 E^2 \sim G_F^2 T^2$

  $G_F \sim 1/m_W^2$, $[G_F] = 1/[energy]^2$
- interaction rate: $\Gamma = n\langle\sigma v\rangle \sim G_F^2 T^5$
- equilibrium condition:

$$\Gamma \sim H \quad \Rightarrow \quad G_F^2 T^5 \sim \frac{T^2}{M_{Pl}}$$

- neutrino decoupling at $T_{dec} \sim (G_F^2 M_{Pl})^{-1/3} \sim 1\,MeV$ ($t \sim 1$ s)

Neutrino temperature

- after decoupling ($T < T_{dec}$) neutrino temperature redshifts with a^{-1}
- at $T \lesssim 0.5$ MeV: e^+e^- annihilations $\rightarrow$ heating of photon plasma
- neutrinos already decoupled $\rightarrow$ do not feel the heating of photons
- $\Rightarrow$ photon temperature decreases slower than neutrino temperature

$$\frac{T_\gamma}{T_\nu} = \left(\frac{11}{4}\right)^{1/3} \approx 1.4 \qquad (T < 1\,MeV)$$

(entropy conservation)

- today: $T_0 = 2.7\,K \;\Rightarrow\; T_{\nu 0} = 1.9\,K$

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Neutrinos today

- today: $T_{\nu 0} = 1.9\,K$
- number density of relic neutrinos today per flavour:

$$n_\nu = \frac{3}{4}\left(\frac{T_{\nu 0}}{T_{\gamma 0}}\right)^3 n_\gamma = \frac{3}{4}\frac{4}{11} n_\gamma \approx 112\,cm^{-3}$$

using $g_\nu = g_\gamma = 2$, $n_\gamma \approx 411\,cm^{-3}$

- $T_{\nu 0} \ll \sqrt{\Delta m^2_{21}} \sim 0.0086\,\text{eV}$, $\sqrt{\Delta m^2_{31}} \sim 0.05\,\text{eV}$
  $\Rightarrow$ at least two neutrinos are non-relativistic today
- energy density for non-rel. neutrinos: $\rho_\nu = n_\nu \sum_i m_i$
- robust upper bound on neutrino mass from requiring $\Omega_\nu < 1$:

$$\Omega_\nu = \frac{\rho_\nu}{\rho_{crit}} \approx \frac{\sum_i m_i}{47\,\text{eV}} \quad \Rightarrow \quad \sum_i m_i < 47\,\text{eV}$$

[much stronger bound from structure formation - see later]

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Outline

Big Bang nucleosynthesis (BBN)

- protons and neutrons in thermal equilibrium till around 1 MeV via

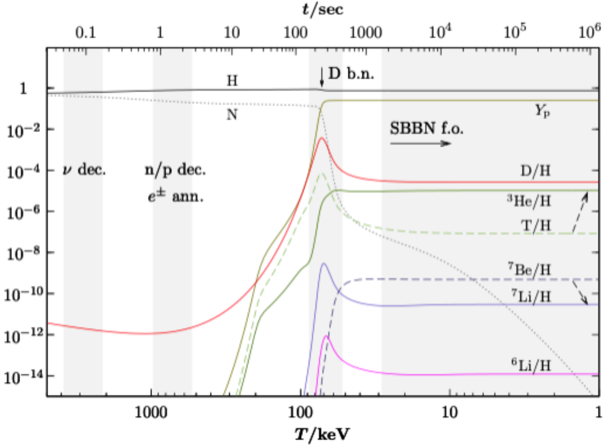
$$n + \nu_e \leftrightarrow p + e^-$$
$$n + e^+ \leftrightarrow p + \bar{\nu}_e$$
$$n \leftrightarrow p + \bar{\nu}_e + e^-$$

- when temperature falls further nucleii start to form:

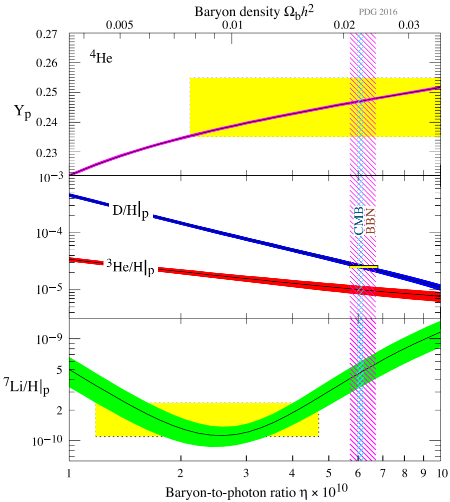
binding energies [MeV]:

D	^{3}H	^{3}He	^{4}He
2.22	8.5	7.7	28.3

- formation of heavier nucleii is suppressed by low D binding energy
- $\sim 10^{10}$ more photons than baryons
  $\rightarrow$ D starts to form only around 0.07 MeV
- final out come of relative abundances sensitively depends on the photon-baryon ratio $\eta \propto \Omega_B h^2$



M. Pospelov, J. Pradler,
Big Bang Nucleosynthesis as a Probe of New Physics
Ann.Rev.Nucl.Part.Sci. 60 (2010) 539 [arXiv:1011.1054]



determinations of the baryon density from Big Bang Nucleosynthesis and CMB are in perfect agreement:

$$\Omega_b h^2 = 0.0214 \pm 0.0020 \quad \text{(BBN)}$$
$$\Omega_b h^2 = 0.0223 \pm 0.0002 \quad \text{(CMB)}$$

Counting neutrino flavours

- neutron/proton abundance in thermal equilibrium:

$$\left.\frac{n_n}{n_p}\right|_{eq} \approx e^{-(m_n - m_p)/T}$$

  with $m_n - m_p \approx 1.3$ MeV $\Rightarrow$ falls exponentially with decreasing T
- neutron/proton ratio gets frozen (up to very slow neutron decay), once above processes fall out of equilibrium, i.e., when

$$\Gamma_{n\leftrightarrow p} < H$$

- as soon as ^{4}He forms, all available neutrons will be bound in ^{4}He $\Rightarrow$ final ^{4}He yield depends sensitively on neutron abundance, i.e., on freeze-out of n/p ratio

Counting neutrino flavours

- neutron/proton ratio gets frozen (up to very slow neutron decay), once above processes fall out of equilibrium, i.e., when

$$\Gamma_{n\leftrightarrow p} < H$$

- remember:

$$H = \sqrt{\frac{8\pi G_N}{3}\sum_{i\in R}\rho_i}$$

$$\sum_{i\in R}\rho_i = \frac{\pi^2}{30}\left[2T_\gamma^4 + 2\frac{7}{8}N_{eff}\,T_\nu^4\right]$$

- ^{4}He abundance depends sensitively on # of neutrino flavours N_{eff}
- similar: also CMB depends on N_{eff} (matter-radiation equality,...)

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$$N_{\rm eff} = 2.99 \pm 0.34\,(95\%)$$

Planck 1807.06209

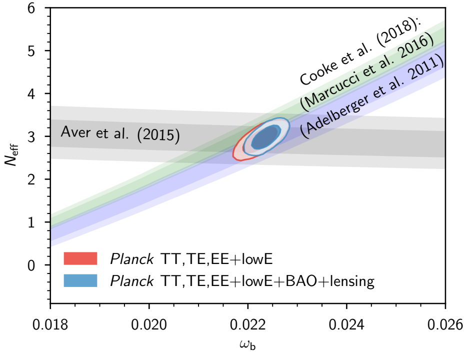

Fig. 39. Constraints in the $\omega_{\rm b}$–$N_{\rm eff}$ plane from *Planck* TT,TE,EE +lowE and *Planck* TT,TE,EE+lowE+BAO+lensing data (68 % and 95 % contours) compared to the predictions of BBN combined with primordial abundance measurements of helium (Aver et al. 2015, in grey) and deuterium (Cooke et al. 2018, in green and blue, depending on which reaction rates are assumed).

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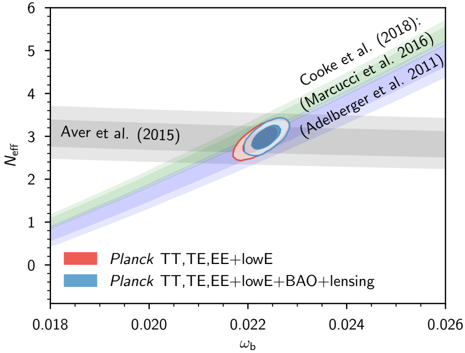
severe constraint for light sterile neutrinos!



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Cosmology vs particle colliders

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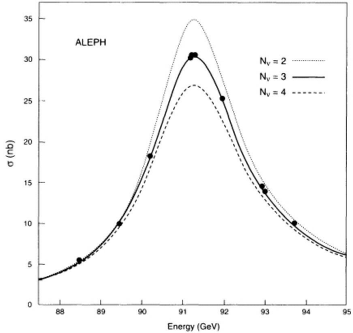

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PDG 2020

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number of relativistic degrees of freedom in thermal equilibrium during BBN and CMB

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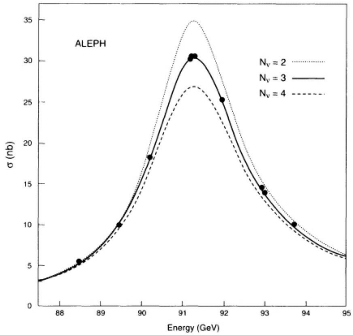

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PDG 2020

number of invisble particles with $2m_{invis} < m_Z$ coupling to the Z^0 boson

Outline

Matter power spectrum

- density fluctuations in the matter density:

$$\delta(\vec{x}, t) = \frac{\rho(\vec{x}, t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

- Fourier transform:

$$\delta_k = \int d^3x \delta(\vec{x}, t) e^{-i\vec{k}\vec{x}}$$

- definition of matter power spectrum:

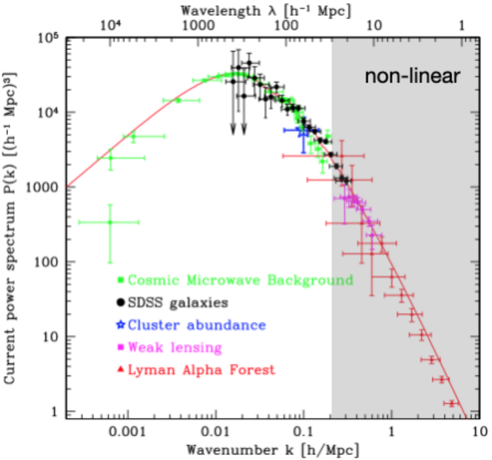
$$\langle \delta_k, \delta_{k'} \rangle = (2\pi)^3 \delta^3(\vec{k} - \vec{k}') P(k)$$

- finite volume: $(2\pi)^3 \delta^3(\vec{k} - \vec{k}') \to \delta_{kk'}$

$$P(k) = \langle |\delta_k|^2 \rangle$$

$\to P(k)$: variance of density fluctuations with wave number $k = 2\pi/\lambda$

Matter power spectrum



The 3-D power spectrum of galaxies from the SDSS
Astrophys.J. 606 (2004) 702-740 [astro-ph/0310725]

Growth of structure for non.-rel. Matter (DM + B)

linearized Einstein equations:

$$\ddot{\delta}_k + 2H\dot{\delta}_k - 4\pi G_N \rho_M \delta_k = 0$$

assume matter domination $\rho_M \propto a^{-3}$, use Friedman $H^2 = \frac{8}{3}\pi G_N \rho_M$, $a \propto t^{2/3}$, and $H = 2/(3t)$:

$$\ddot{\delta}_k + \frac{4}{3t}\dot{\delta}_k - \frac{2}{3t^2}\delta_k = 0$$

solution: $\delta_k \propto t^{2/3} \propto a$

$\Rightarrow$ linear growth of matter fluctuations during matter domination (indep. of k)
$\Rightarrow$ cosmic structure can form

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Effect of neutrinos on structure formation

- relativistic non-interacting fluid does not cluster
- free-streaming length λ_{FS}:

$$\lambda_{FS} \sim \frac{c}{H(t)}\,, \qquad k_{FS} = \sqrt{\frac{3}{2}}\frac{H(t)}{c}$$

- in matter domination: $k_{FS} \propto 1/t$
- neutrinos become non-relativistic when $3T \approx \langle p \rangle < m_\nu$
  at that point $v \to 0$, $\lambda_{FS} \to 0$, $k_{FS} \to \infty$
- λ_{FS} has a maximum when $m_\nu \sim 3T \Rightarrow$ define k_{NR} for $m_\nu \approx 3T$:

$$k_{NR} \approx 0.01\,\mathrm{Mpc}^{-1}\sqrt{\frac{m_\nu}{\mathrm{eV}}}$$

$k < k_{NR}$: neutrinos behave as dark matter: $\Omega_M \to \Omega_M + \Omega_\nu$
$k > k_{NR}$: neutrinos suppress structure formation due to free-streaming

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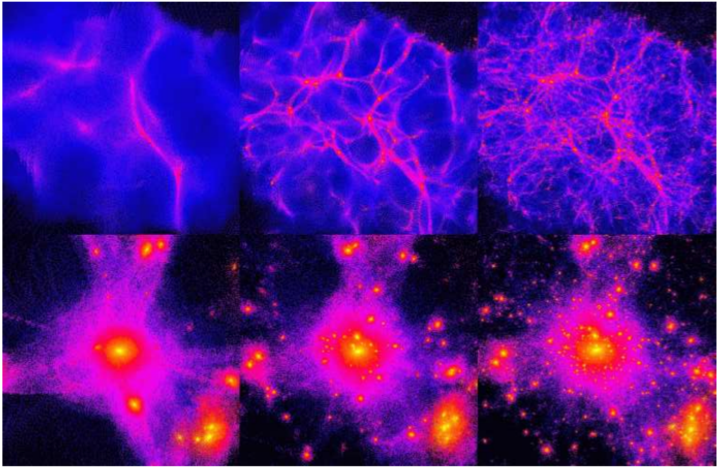
$$\lambda_{FS} \sim \frac{c}{H(t)}\,, \qquad k_{FS} = \sqrt{\frac{3}{2}}\frac{H(t)}{c}$$

- in matter domination: $k_{FS} \propto 1/t$
- neutrinos become non-relativistic when $3T \approx \langle p \rangle < m_\nu$
  at that point $v \to 0$, $\lambda_{FS} \to 0$, $k_{FS} \to \infty$
- λ_{FS} has a maximum when $m_\nu \sim 3T \Rightarrow$ define k_{NR} for $m_\nu \approx 3T$:

$$k_{NR} \approx 0.01\,\mathrm{Mpc}^{-1}\sqrt{\frac{m_\nu}{\mathrm{eV}}}$$

$k < k_{NR}$: neutrinos behave as dark matter: $\Omega_M \to \Omega_M + \Omega_\nu$
$k > k_{NR}$: neutrinos suppress structure formation due to free-streaming



Ben Moore simulations

Consider modes with $k > k_{NR}$

define neutrino fraction: $\rho_{tot} = \rho_M + \rho_\nu = \rho_M(1+f_\nu)$ with $f_\nu \ll 1$

1. normalization effect on power spectrum:

$$P(k) \approx \left\langle \left| \frac{\delta\rho_M + \delta\rho_\nu}{\rho_{tot}} \right|^2 \right\rangle \approx \frac{1}{(1+f_\nu)^2} \left\langle \left| \frac{\delta\rho_M}{\rho_M} \right|^2 \right\rangle \approx (1-2f_\nu)\langle|\delta_M|^2\rangle$$

   since for $k > k_{NR}$ we have $\delta\rho_M \gg \delta\rho_\nu$

2. suppression of structure growth: $\ddot{\delta}_M + 2H\dot{\delta}_M - 4\pi G_N \rho_M \delta_M = 0$

   using now Friedman $H^2 = \frac{8}{3}\pi G_N \rho_{tot} = \frac{8}{3}\pi G_N (1+f_\nu)\rho_M$

$$\ddot{\delta}_M + \frac{4}{3t}\dot{\delta}_M - \frac{2}{3t^2}(1-f_\nu)\delta_M = 0$$

solution: $\delta_M \propto t^{\frac{2}{3}\left(1-\frac{3}{5}f_\nu\right)} \propto a^{1-\frac{2}{5}f_\nu}$

1. + 2. numerical fit: $\frac{\Delta P(k)}{P(k)} \approx -8f_\nu \approx -\left(\frac{m_\nu}{\text{eV}}\right) \qquad k > k_N$

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Effect of neutrinos on matter power spectrum

Lesgourgues, Pastor, astro-ph/0603494

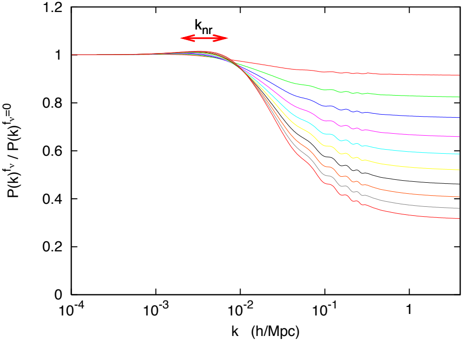
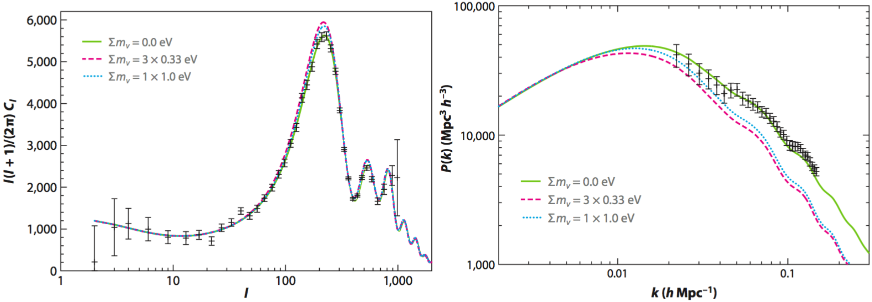


Fig. 13. Ratio of the matter power spectrum including three degenerate massive neutrinos with density fraction f_ν to that with three massless neutrinos. The parameters $(\omega_{\rm m}, \Omega_\Lambda) = (0.147, 0.70)$ are kept fixed, and from top to bottom the curves correspond to $f_\nu = 0.01, 0.02, 0.03, \ldots, 0.10$. The individual masses m_ν range from 0.046 eV to 0.46 eV, and the scale $k_{\rm nr}$ from $2.1 \times 10^{-3} h\,{\rm Mpc}^{-1}$ to $6.7 \times 10^{-3} h\,{\rm Mpc}^{-1}$ as shown on the top of the figure. $k_{\rm eq}$ is approximately equal to $1.5 \times 10^{-2} h\,{\rm Mpc}^{-1}$.

here: $\omega_i \equiv \Omega_i h^2$, $3m_\nu \equiv \sum_i m_i$, $f_\nu \equiv \omega_\nu/\omega_m = 3m_\nu/(\omega_m \times 93\,\mathrm{eV})$

Effect of neutrino mass on CMB and LSS



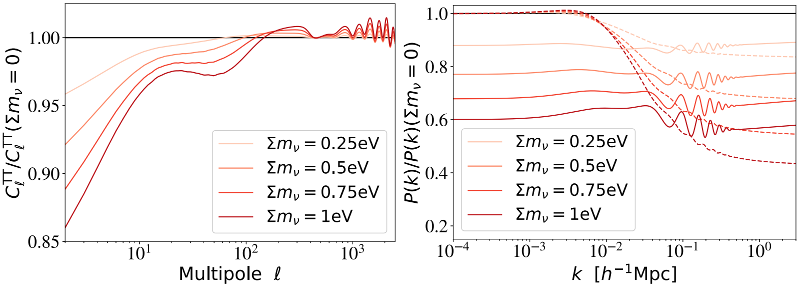
data points: WMAP 3yr and 2dF '05

Y.Y.Y. Wong, 1111.1436

- CMB: mainly height of 1st peak
- LSS: suppression of structure at scales smaller than 1–10 Mpc
- effects correlated with other parameters of the ΛCDM model

Effect of neutrino mass on CMB and LSS

Lesgourgues, Verde, PDG 2020



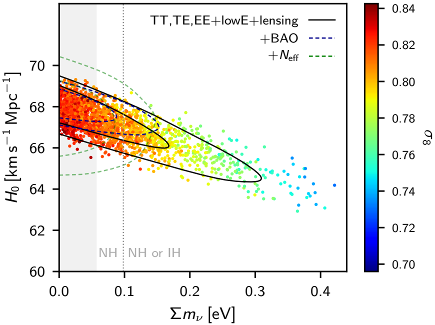
- dashed: fixing $\Omega_M = \Omega_B + \Omega_{CDM} + \Omega_\nu \to$ modify z_{eq} when changing m_ν $\to$ strong effect on CMB
- solid: fixing Ω_B, Ω_{CDM}, angular scale of sound hor.,..., such that $z_{eq} \approx const$, minimizing effect on CMB
  $\to$ (nearly) scale invariant suppression of $P(k)$; correlation of m_ν with H_0

Neutrino mass bound from cosmology

$$\sum m_\nu < 0.24\,\text{eV (CMB)}$$
$$\sum m_\nu < 0.12\,\text{eV (CMB+BAO)}$$

limits at 95% CL

Planck 1807.06209



Neutrino mass bound from cosmology

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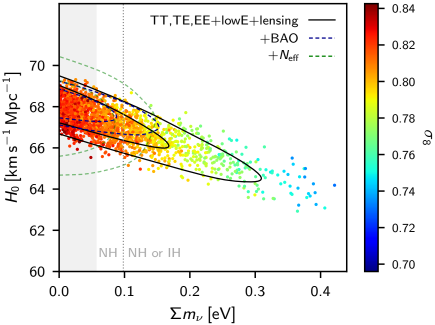


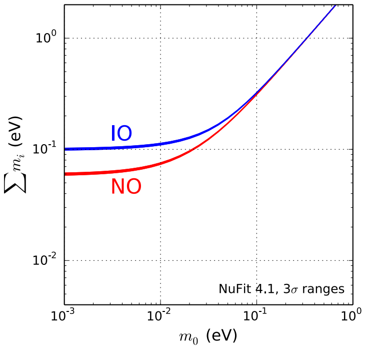
- currently strongest bounds on absolute neutrino mass (see later)
- severe constraint for light sterile neutrinos
- rather stable wrt to modifications of cosmology

Cosmology is sensitive to the sum of neutrino masses

$$\sum_{i=1}^{3} m_i = \begin{cases} m_0 + \sqrt{\Delta m^2_{21} + m_0^2} + \sqrt{\Delta m^2_{31} + m_0^2} & \text{(NO)} \\ m_0 + \sqrt{|\Delta m^2_{32}| + m_0^2} + \sqrt{|\Delta m^2_{32}| - \Delta m^2_{21} + m_0^2} & \text{(IO)} \end{cases}$$

minimum values for $m_0 = 0$:

$$\sum m_i \Big|_{\min} = \begin{cases} 58.5 \pm 0.48\,\text{meV} & \text{(NO)} \\ 98.6 \pm 0.85\,\text{meV} & \text{(IO)} \end{cases}$$



Cosmology is sensitive to the sum of neutrino masses

$$\sum_{i=1}^{3} m_i = \begin{cases} m_0 + \sqrt{\Delta m_{21}^2 + m_0^2} + \sqrt{\Delta m_{31}^2 + m_0^2} & \text{(NO)} \\ m_0 + \sqrt{|\Delta m_{32}^2| + m_0^2} + \sqrt{|\Delta m_{32}^2| - \Delta m_{21}^2 + m_0^2} & \text{(IO)} \end{cases}$$

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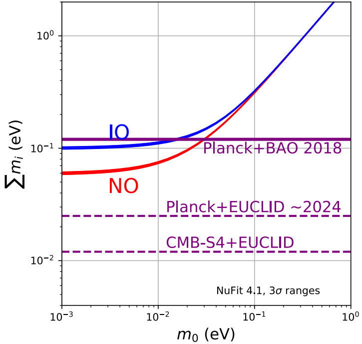
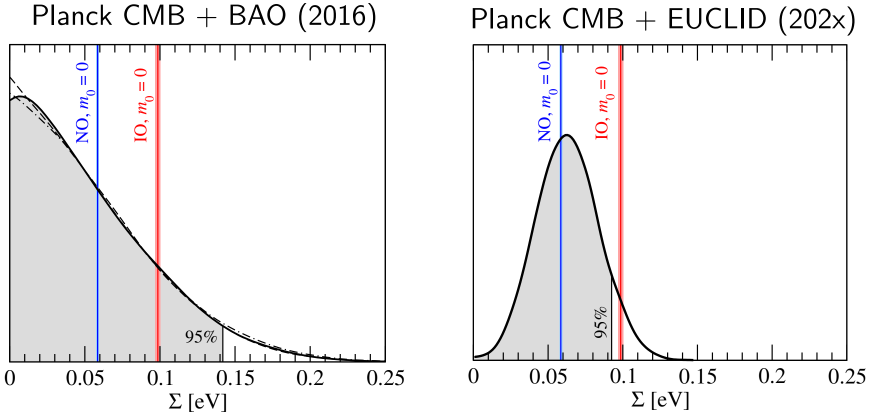
- current limit close to IO minimum
- detection of non-zero neutrino mass expected soon!



Excluding IO with cosmology?



Hannestad, Schwetz, 2016

Relaxing the neutrino mass bound with neutrino decay

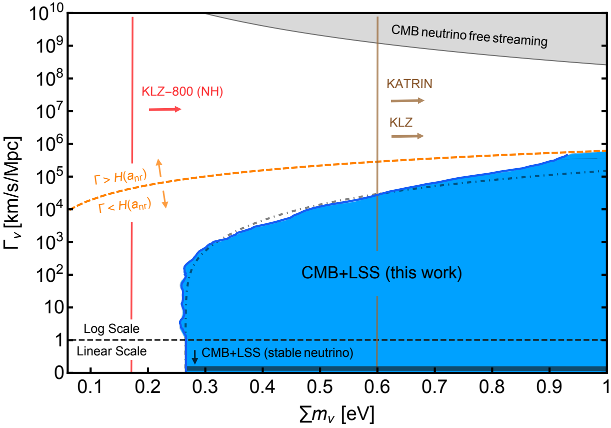
Chacko, Dev, Du, Poulin, Tsai,
1909.05275, 2002.08401

assume neutrino decay:

$$\nu_i \to \nu_4 \phi$$

with $i = 1, 2, 3$ and
$m_4, m_\phi = 0$

decay rate: Γ_ν



see also Escudero,Fairbairn,1907.05425; Escudero,Lopez-Pavon,Rius,Sandner,2007.04994

Outline

Summary

- Neutrinos play an important role in cosmology:
  - control the formation of light elements (BBN)
  - control the formation of cosmic structure
  - many more, not discussed here
- Cosmology is a powerful tool to constrain neutrino properties:
  - number of neutrino flavours
  - stringent bound on sum of neutrino masses
    detection of non-zero neutrino mass is in reach
  - can constrain non-standard neutrino properties, e.g., sterile neutrinos, neutrino decay, neutrino self-interactions,... (many more effects not discussed here)