KSETA topical courses

Neutrino physics I: Neutrino Oscillations

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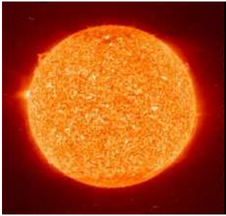
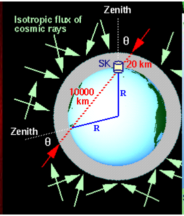
KIT
Karlsruher Institut für Technologie

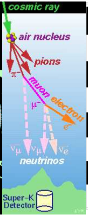
HIDDeν
Hunting Invisibles: Dark sectors,
Dark matter and Neutrinos

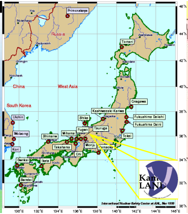
Karlsruhe, 7-8 Oct 2020

Neutrinos oscillate...









... and have mass ⇒ physics beyond the Standard Model

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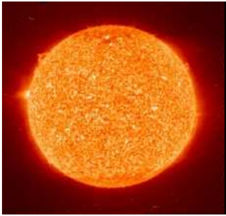

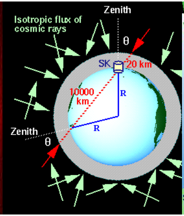

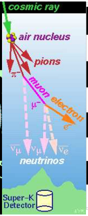

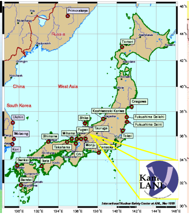

... and have mass ⇒ physics beyond the Standard Model

- Lecture I: Neutrino Oscillations
- Lecture II: Neutrinos in Cosmology
- Lecture III: Neutrino mass - Dirac versus Majorana
- Lecture IV: Neutrinos and physics beyond the Standard Model

Literature

- Phenomenology:
  C. Giunti, C.W. Kim: Fundamentals of Neutrino Physics and Astrophysics
- Neutrino Cosmology:
  Lesgourgues, Mangano, Miele, Pastor, Neutrino Cosmology (2013, Cambridge Univ. Press)
  Lesgourgues, Pastor, astro-ph/06034494
- Theory aspects:
  R.N. Mohapatra, P.B. Pal, Massive Neutrinos In Physics And Astrophysics (1998, World Scientific Publishing)
- more literature during the lectures

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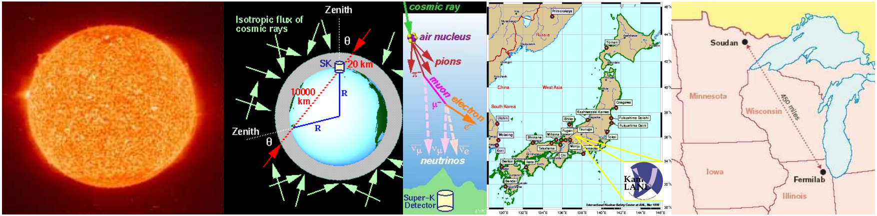

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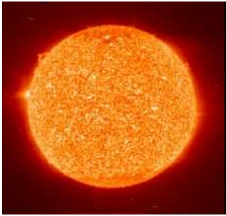

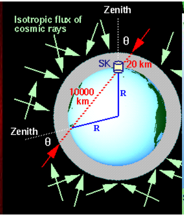

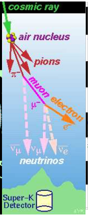

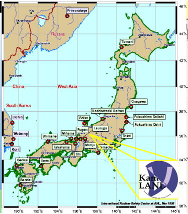

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Outline - Neutrino Physics I

Outline

Introduction
- Historical remarks

Lepton mixing

Neutrino oscillations
- Oscillations in vacuum
- Oscillations in matter
- Varying matter density and MSW

Global data and 3-flavour oscillations
- Qualitative picture
- Global analysis
- Oscillations – outlook

Summary - neutrino oscillations

Neutrinos ...

- are particles with very small mass:

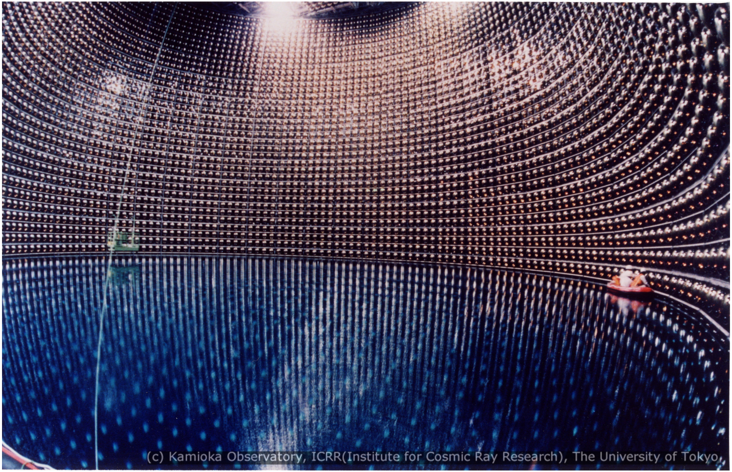
$$m_{\text{neutrino}} \lesssim 1\,\text{eV} \sim 10^{-6}\, m_{\text{electron}}$$

- are the only known electrically neutral fermions
  participate only in weak interaction and gravitation

- most abundant fermion in the Universe
  336 cosmic neutrinos/cm^3 (comparable to 411 CMB photons/cm^3)

How “weak” are weak interactions?

Super Kamiokande: 50 000 t water



(c) Kamioka Observatory, ICRR(Institute for Cosmic Ray Research), The University of Tokyo

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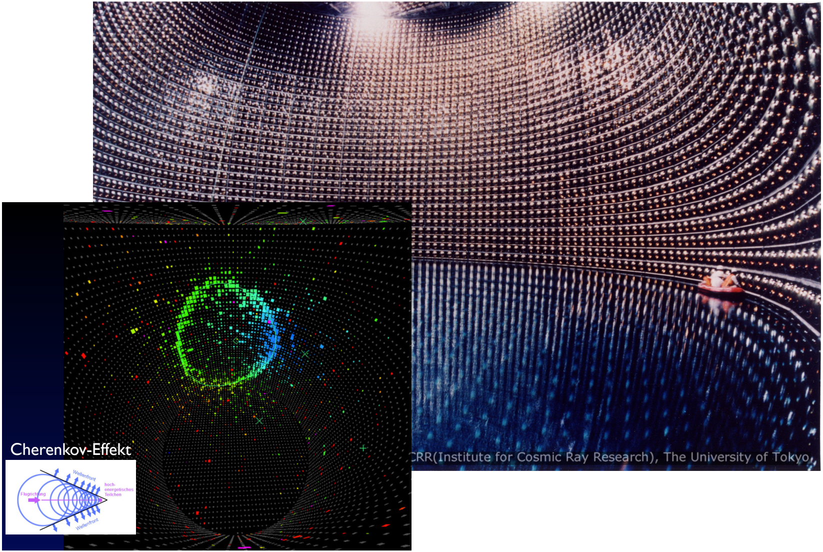

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- Every second 10^{18} solar neutrinos are passing through the SuperKamiokande detector.
- Only 14 neutrinos per day are detected.

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out of the 10^{18} neutrinos/s only 10^{14} are energetic enough to be seen by SuperK (^{8}B neutrinos) $\quad\rightarrow\quad$ 10^{19} ^{8}B neutrinos/day

“detection efficiency” of $14/10^{19} \simeq 10^{-18}$ (!)

Despite their weak interaction ...

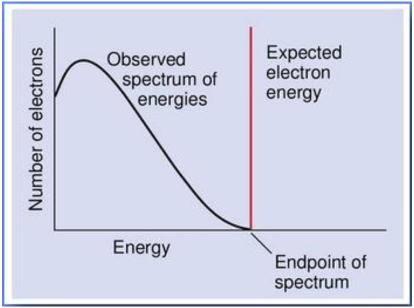
... neutrinos are crucial for our existence:

- control the formation of light elements ∼15min after the Big Bang
- control the formation of heavy elements in supernova explosions
- control the formation of structure in the Universe (galaxies)
- play a crucial role for the shining of the Sun
- may be responsible for creating an access of matter over antimatter in the very early Universe (Leptongenesis)

Historical remarks

1930: missing energy in nuclear beta decay

$$(A, Z) \to (A, Z+1) + e^-$$



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1930: missing energy in nuclear beta decay

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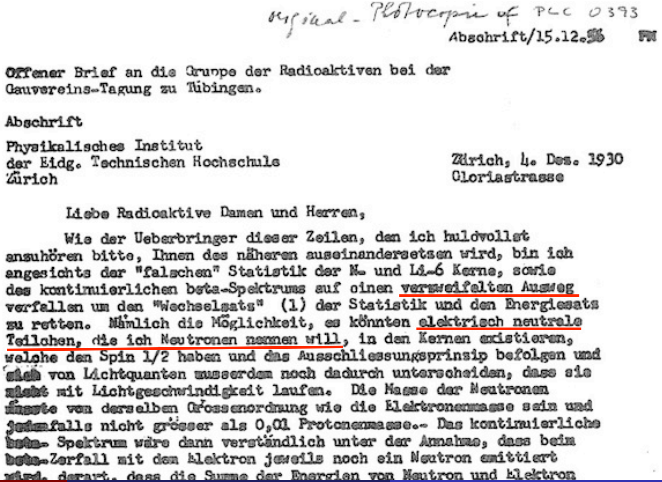
Wolfgang Pauli
(1900-1958)



original - Photocopie of PLC 0393
Abschrift/15.12.56 [illegible]

Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron

Historical remarks

- 1930: Wolfgang Pauli postulates "neutron" to save energy and angular momentum conservation in nuclear beta decay
- 1933: Enrico Fermi develops theory of beta decay and introduces the name "neutrino"
- 1956: first detection of neutrinos at a nuclear reactor by Frederick Reines and Clyde Cowan
- 1956/57: Wu demonstrates parity violation in weak interactions and Goldhaber shows that the neutrino is left-handed
- 1962: discovery of second type of neutrino by Steinberger, Schwartz and Lederman
- 1969: detection of solar neutrinos by Ray Davis
- 1989: LEP determines the number of neutrino flavours coupling to the Z^0 to be 2.9840 ± 0.0082
- 1998, 2002: discovery of neutrino oscillations by the SuperKamiokande, SNO, KamLAND experiments proves that neutrinos have mass

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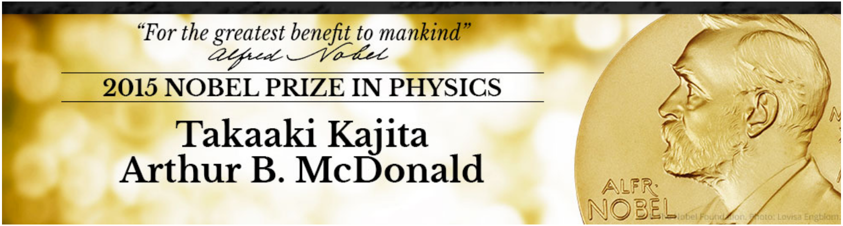

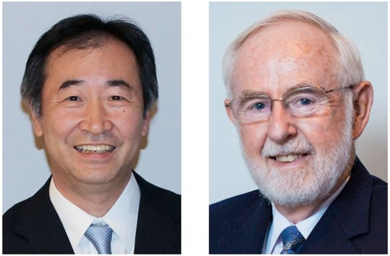

„…for the discovery of neutrino oscillations, which shows that neutrinos have mass“

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Lepton mixing

Neutrino oscillations

- Oscillations in vacuum
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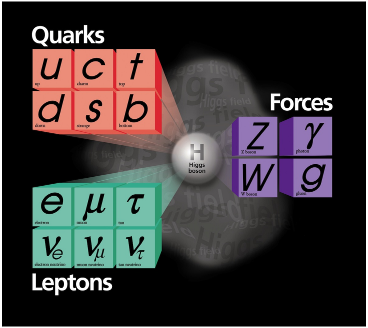
Summary - neutrino oscillations

The Standard Model



Fermions in the Standard Model come in three generations (“Flavours”)

Neutrinos are the “partners” of the charged leptons

more precisely: they form a doublet under the SU(2) gauge symmetry

Flavour neutrinos

A neutrino of flavour α is defined by the charged current interaction with the corresponding charged lepton:

$$\mathcal{L}_{\mathrm{CC}} = -\frac{g}{\sqrt{2}} W^\rho \sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L} \gamma_\rho \ell_{\alpha L} + \mathrm{h.c.}$$

for example

$$\pi^+ \to \mu^+ \nu_\mu$$

the muon neutrino ν_μ comes together with the charged muon μ^+

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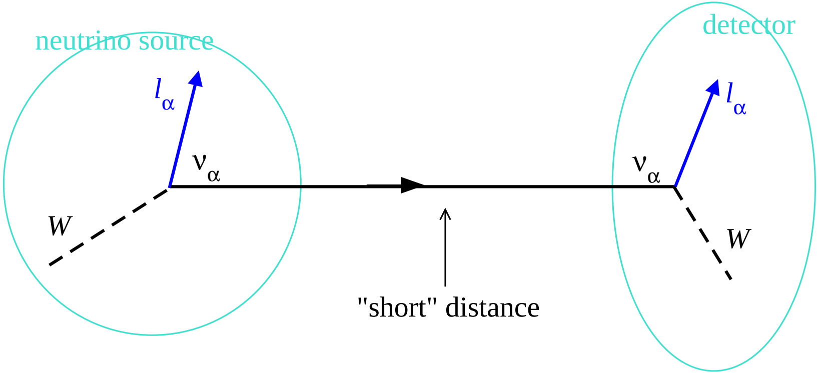

Let's give mass to the neutrinos

Majorana mass term:

$$\mathcal{L}_{\mathrm{M}} = -\frac{1}{2} \sum_{\alpha,\beta=e,\mu,\tau} \nu_{\alpha L}^{T} C^{-1} \mathcal{M}_{\alpha\beta} \nu_{\beta L} + \mathrm{h.c.}$$

$\mathcal{M}$: symmetric mass matrix

In the basis where the CC interaction is diagonal the mass matrix is in general not a diagonal matrix

any complex symmetric matrix $\mathcal{M}$ can be diagonalised by a unitary matrix

$$U_\nu^T \mathcal{M} U_\nu = m \,, \qquad m : \text{ diagonal, } m_i \geq 0$$

Lepton mixing

$$\begin{aligned}\mathcal{L}_{\rm CC} &= -\frac{g}{\sqrt{2}} W^\rho \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^{3} \bar{\nu}_{iL} U^*_{\alpha i} \gamma_\rho \ell_{\alpha L} + \mathrm{h.c.} \\ \mathcal{L}_{\rm M} &= -\frac{1}{2} \sum_{i=1}^{3} \nu_{iL}^T C^{-1} \nu_{iL} m_i^\nu - \sum_{\alpha=e,\mu,\tau} \bar{\ell}_{\alpha R} \ell_{\alpha L} m_\alpha^\ell + \mathrm{h.c.}\end{aligned}$$

Pontecorvo-Maki-Nakagawa-Sakata lepton mixing matrix:

$$(U_{\alpha i}) \equiv U_{\rm PMNS}$$

Lepton mixing

- Flavour neutrinos ν_α are superpositions of massive neutrinos ν_i:

$$\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i}\nu_i \qquad (\alpha = e, \mu, \tau)$$

- mismatch between mass and interaction basis
- Example for two neutrinos:

$$\begin{aligned} \nu_e &= \cos\theta\,\nu_1 + \sin\theta\,\nu_2 \\ \nu_\mu &= -\sin\theta\,\nu_1 + \cos\theta\,\nu_2 \end{aligned}$$

- The same phenomenon happens also for quarks (CKM matrix)

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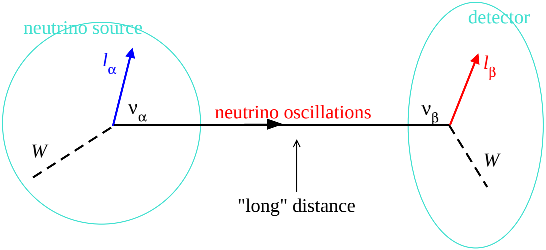
Neutrino oscillations



$|\nu_\alpha\rangle = U^*_{\alpha i}|\nu_i\rangle$ $\quad e^{-i(E_i t - p_i x)}$ $\quad |\nu_\beta\rangle = U^*_{\beta i}|\nu_i\rangle$

oscillation amplitude:

$$\begin{aligned}\mathcal{A}_{\nu_\alpha\to\nu_\beta} &= \langle\nu_\beta| \quad \text{propagation} \quad |\nu_\alpha\rangle \\ &= \sum_{i,j} U_{\beta j}\langle\nu_j| \quad e^{-i(E_i t - p_i x)} \quad |\nu_i\rangle U^*_{\alpha i} = \sum_i U_{\beta i}U^*_{\alpha i}e^{-i(E_i t - p_i x)}\end{aligned}$$

Neutrino oscillations in vacuum

oscillation amplitude:

$$\mathcal{A}_{\nu_\alpha \to \nu_\beta} = \sum_i U_{\beta i} U^*_{\alpha i} e^{-i(E_i t - p_i x)} \quad \to \quad P_{\nu_\alpha \to \nu_\beta} = \left| \mathcal{A}_{\nu_\alpha \to \nu_\beta} \right|^2$$

need to calculate phase differences:

$$\phi_{ji} = (E_j - E_i)t - (p_j - p_i)x \quad \text{with} \quad E_i^2 = p_i^2 + m_i^2$$

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after some hand waving:

$$\phi_{ji} \approx \frac{\Delta m^2_{ji} L}{2\,E} \quad \text{with} \quad \Delta m^2_{ji} \equiv m_j^2 - m_i^2$$

A hand-waving derivation for two flavours

oscillation phase:

$$\phi = (E_2 - E_1)t - (p_2 - p_1)x \quad \text{with} \quad E_i^2 = p_i^2 + m_i^2$$

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define: $\Delta E = E_2 - E_1, \quad \Delta E^2 = E_2^2 - E_1^2, \quad \bar{E} = (E_1 + E_2)/2$

then: $\Delta E^2 = 2\bar{E}\Delta E$ (similar for p and m)

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$$\begin{aligned} \phi &= \Delta E t - \frac{\Delta p^2}{2\bar{p}} x = \Delta E t - \frac{\Delta E^2 - \Delta m^2}{2\bar{p}} x \\ &= \Delta E t - \frac{2\bar{E}}{2\bar{p}} \Delta E x + \frac{\Delta m^2}{2\bar{p}} x \end{aligned}$$

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$$\begin{aligned} \phi &= \Delta E t - \frac{\Delta p^2}{2\bar{p}}x = \Delta E t - \frac{\Delta E^2 - \Delta m^2}{2\bar{p}}x \\ &= \Delta E t - \frac{2\bar{E}}{2\bar{p}}\Delta E x + \frac{\Delta m^2}{2\bar{p}}x \end{aligned}$$

use “average velocity” of the neutrino $v = \bar{p}/\bar{E}$ and $x \approx vt$:

$$\boxed{\phi \approx \frac{\Delta m^2}{2\bar{p}}x \approx \frac{\Delta m^2}{2\bar{E}}x}$$

problematic at least for the following reasons:

- use of average velocity is arbitrary (derivations in the literature use "equal momentum" or "equal energy")
- assuming $x \approx vt$ is inconsistent with plain wave ansatz for neutrino propagation $\propto e^{-i(E_i t - p_i x)}$

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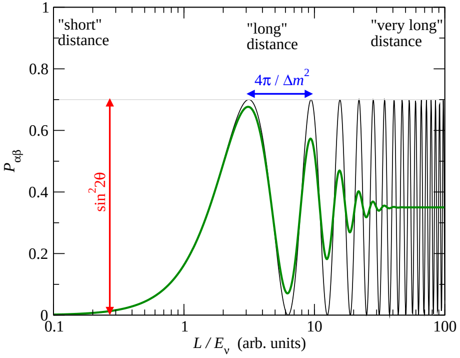
To derive the oscillation probability rigorously one needs either a wave-packet treatment or field theory
e.g., Akhmedov, Kopp, JHEP 1004:008 (2010) [1001.4815]

2-neutrino oscillations

Two-flavour limit:

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \qquad P = \sin^2 2\theta \, \sin^2 \frac{\Delta m^2 L}{4E_\nu}$$

oscillations are sensitive to mass differences (not absolute masses)



$$\frac{\Delta m^2 L}{4E_\nu} = 1.27 \frac{\Delta m^2[\mathrm{eV}^2]\, L[\mathrm{km}]}{E_\nu[\mathrm{GeV}]}$$

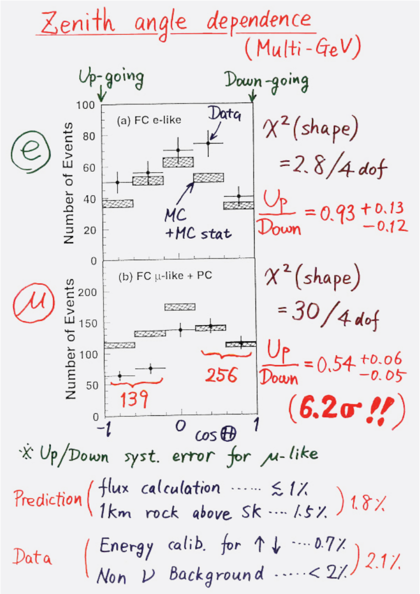
Neutrinos oscillate!

1998: SuperKamiokande
atmospheric neutrinos

- zenith-angle dependent deficit of multi-GeV μ-like events
- consistent with $\nu_\mu \to \nu_\tau$ oscillations with

  $\Delta m^2 \simeq 2.5 \times 10^{-3}$ eV^2
  $\sin^2 2\theta \simeq 1$

Nobel prize 2015
Takaaki Kajita



Zenith angle dependence
(Multi-GeV)

χ^2(shape) $= 2.8/4$ dof

$\frac{\text{Up}}{\text{Down}} = 0.93\,^{+0.13}_{-0.12}$

χ^2(shape) $= 30/4$ dof

$\frac{\text{Up}}{\text{Down}} = 0.54\,^{+0.06}_{-0.05}$

(6.2σ!!)

※ Up/Down syst. error for μ-like

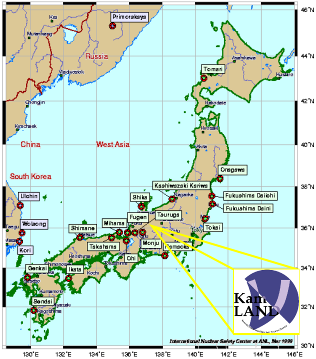
Prediction (flux calculation …… ≲ 1%
1km rock above SK ···· 1.5%) 1.8%

Data (Energy calib. for ↑↓ ···· 0.7%
Non ν Background ····· < 2%) 2.1%

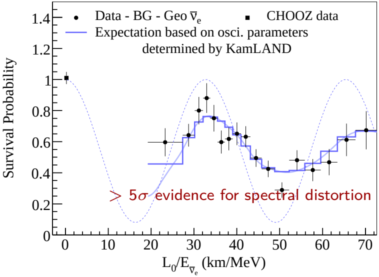
Neutrinos oscillate!

$$P_{\text{survival}} \approx 1 - \sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2}{4}\frac{L}{E_\nu}\right)$$

KamLAND $\bar{\nu}_e \to \bar{\nu}_e$



$\langle L \rangle \sim 180$ km



$> 5\sigma$ evidence for spectral distortion

Neutrinos oscillate!

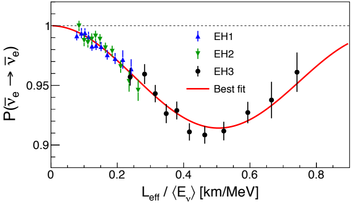
$$P_{\text{survival}} \approx 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2}{4} \frac{L}{E_\nu} \right)$$

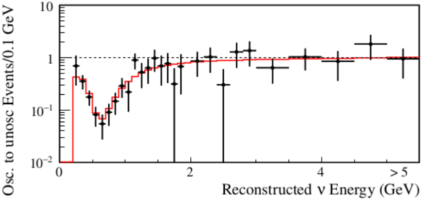
DayaBay, 2015

$\bar{\nu}_e \to \bar{\nu}_e$, $\langle L \rangle \sim 2$ km

T2K, 2015

$\nu_\mu \to \nu_\mu$, $\langle L \rangle \sim 295$ km





The matter effect

When neutrinos pass through matter the SM interactions with the particles in the background induce an effective potential for the neutrinos

Effective 4-point interaction Hamiltonian

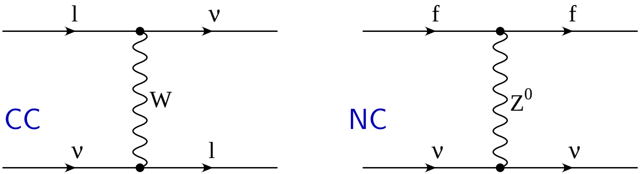
$$H_{\rm int}^{\nu_\alpha} = \frac{G_F}{\sqrt{2}}\,\bar\nu_\alpha\gamma_\mu(1-\gamma_5)\nu_\alpha \underbrace{\sum_f \bar f\gamma^\mu(g_V^{\alpha,f} - g_A^{\alpha,f}\gamma_5)f}_{J^\mu_{\rm mat}}$$

coherent forward scattering amplitude leads to an "index of refraction"
→ proportional to G_F! (not G_F^2)

L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); *ibid.* D **20**, 2634 (1979)

Effective matter potential

$$V_{\mathrm{mat}} = \sqrt{2} G_F \,\mathrm{diag}\left(N_e - N_n/2, -N_n/2, -N_n/2\right)$$



- only ν_e feel CC (there are no μ, τ in normal matter)
- NC is the same for all flavours ⇒ potential proportional to identiy has no effect on the evolution
- NC has no effect for 3-flavour active neutrinos, but is important in the presence of sterile neutrinos

Effective Schrödinger equation in matter

$$i\frac{d}{dt}\begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix} = H \begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix}$$

where

$$H = \underbrace{U\mathrm{diag}\left(0, \frac{\Delta m^2_{21}}{2E_\nu}, \frac{\Delta m^2_{31}}{2E_\nu}\right) U^\dagger}_{\text{vaccum}}$$

Effective Schrödinger equation in matter

$$i\frac{d}{dt}\left(\begin{array}{c} a_e \\ a_\mu \\ a_\tau \end{array}\right) = H \left(\begin{array}{c} a_e \\ a_\mu \\ a_\tau \end{array}\right)$$

where

$$H = \underbrace{U \mathrm{diag}\left(0, \frac{\Delta m^2_{21}}{2E_\nu}, \frac{\Delta m^2_{31}}{2E_\nu}\right) U^\dagger}_{\text{vaccum}} + \underbrace{\mathrm{diag}(\sqrt{2} G_F N_e, 0, 0)}_{\text{matter}}$$

$N_e(x)$: electron density along the neutrino path

Effective Schrödinger equation in matter

$$i\frac{d}{dt}\begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix} = H \begin{pmatrix} a_e \\ a_\mu \\ a_\tau \end{pmatrix}$$

where

$$H = \underbrace{U\mathrm{diag}\left(0, \frac{\Delta m^2_{21}}{2E_\nu}, \frac{\Delta m^2_{31}}{2E_\nu}\right) U^\dagger}_{\text{vaccum}} + \underbrace{\mathrm{diag}(\sqrt{2}G_F N_e, 0, 0)}_{\text{matter}}$$

$N_e(x)$: electron density along the neutrino path

for non-constant matter: $H(t)$ → time-dependent Schrödinger eq.
“MSW resonance” Mikheev, Smirnov, Sov. J. Nucl. Phys. 42, 913 (1985)

Neutrino oscillations in constant matter

diagonalize the Hamiltonian in matter:

$$\begin{aligned} H^{\nu}_{\mathrm{mat}} &= U \mathrm{diag}\left(0, \frac{\Delta m^2_{21}}{2E_\nu}, \frac{\Delta m^2_{31}}{2E_\nu}\right) U^\dagger + \mathrm{diag}(\sqrt{2} G_F N_e, 0, 0) \\ &= U_m \mathrm{diag}\left(\lambda_1, \lambda_2, \lambda_3\right) U_m^\dagger \end{aligned}$$

Same expression for oscillation probability, but replace “vacuum” parameters by “matter” parameters

2-neutrino oscillations in constant matter

Two-flavour case:

$$P_{\mathrm{mat}} = \sin^2 2\theta_{\mathrm{mat}} \, \sin^2 \frac{\Delta m^2_{\mathrm{mat}} L}{4E}$$

with

$$\sin^2 2\theta_{\mathrm{mat}} = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

$$\Delta m^2_{\mathrm{mat}} = \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta - A)^2}$$

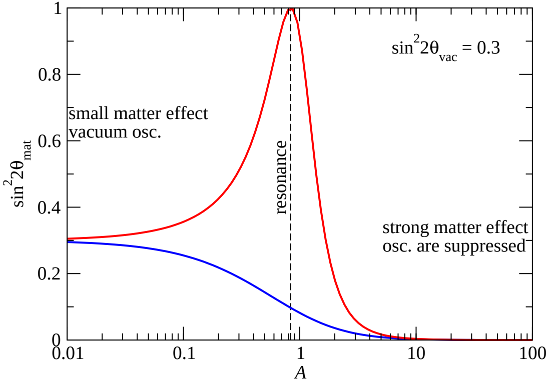
$$A \equiv \frac{2EV}{\Delta m^2}$$

2-neutrino oscillations in constant matter

$$\sin^2 2\theta_{\rm mat} = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - A)^2} \qquad A \equiv \frac{2EV}{\Delta m^2}$$

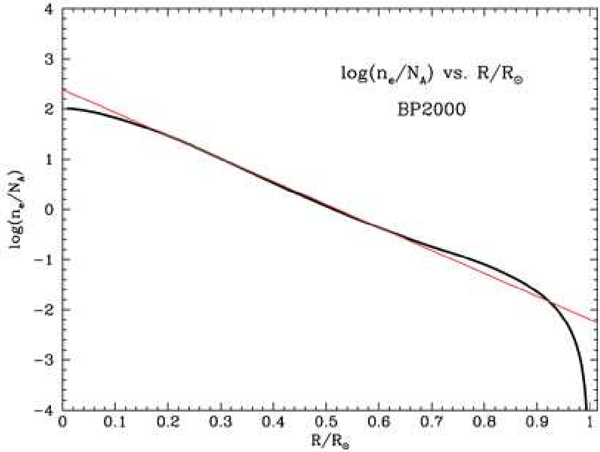
resonance for $\cos 2\theta = A$: “MSW resonance”

Mikheev, Smirnov, Sov. J. Nucl. Phys. 42, 913 (1985)



Varying matter density: example solar neutrinos

The electron density in the sun:



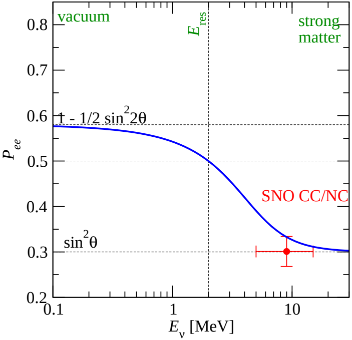
Solar neutrinos and the Sudbury Neutrino Observatory

2002: SNO: CC to NC ratio of solar neutrino flux

CC: $\nu_e + d \to p + p + e^-$
NC: $\nu_x + d \to p + n + \nu_x$

- evidence for $\nu_e \to \nu_\mu, \nu_\tau$ conversion
- MSW effect inside the sun adiabatic conversion through resonance

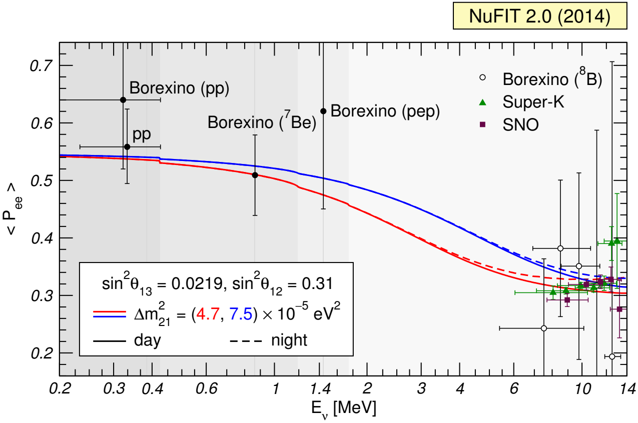
Nobel prize 2015
Art McDonald



$$P_{ee} = \frac{\phi_e}{\phi_e + \phi_\mu + \phi_\tau} = \frac{\phi_{\rm CC}}{\phi_{\rm NC}}$$

Evidence for LMA-MSW

solar neutrino experiments Homestake, SAGE+GNO, Super-K, SNO, Borexino



- $\sin^2\theta < 0.5$ is strong evidence for MSW conversion
- for energies above resonance: $P_{ee} \approx \sin^2\theta$ → best determination of θ_{12}

Outline

3-flavour neutrino parameters

- 3 masses: Δm^2_{21}, Δm^2_{31}, m_0
- 3 mixing angles: θ_{12}, θ_{13}, θ_{23}
- 3 phases: 1 Dirac (δ), 2 Majorana (α_1, α_2)

neutrino oscillations
absolute mass observables
lepton-number violation (neutrinoless double-beta decay)

3-flavour oscillation parameters

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\mathrm{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3-flavour oscillation parameters

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Δm^2_{31} Δm^2_{21}

$$\mathrm{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atm+LBL(dis) react+LBL(app) solar+KamLAND

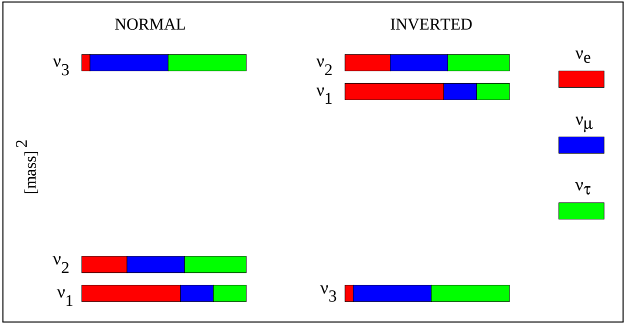
3-flavour effects are suppressed: $\Delta m^2_{21} \ll \Delta m^2_{31}$ and $\theta_{13} \ll 1$ $(U_{e3} = s_{13} e^{-i\delta})$

$\Rightarrow$ dominant oscillations are well described by effective two-flavour oscillations

$\Rightarrow$ present data is already sensitive to sub-leading effects

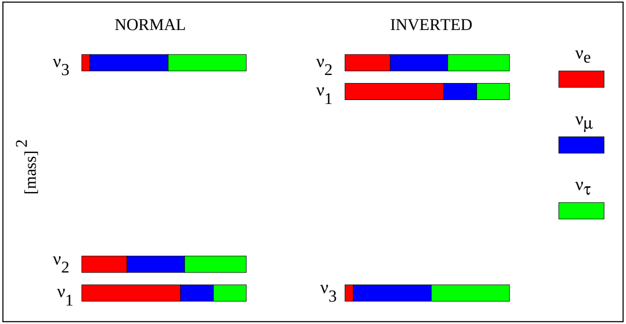
$\Rightarrow$ CP-violation is suppressed by θ_{13}

What we know – masses



- The two mass-squared differences are separated roughly by a factor 30:
  $\Delta m^2_{21} \approx 7 \times 10^{-5} \text{eV}^2\,, \quad |\Delta m^2_{31}| \approx |\Delta m^2_{32}| \approx 2.4 \times 10^{-3} \text{eV}^2$
- at least two neutrinos are massive

Physical interpretation of mixing angles



$\sin\theta_{13} = |U_{e3}|$ (ν_e component in ν_3) = (ν_3 component in ν_e)

$\tan\theta_{12} = \frac{|U_{e2}|}{|U_{e1}|}$ ratio of ν_2 and ν_1 component in ν_e

$\tan\theta_{23} = \frac{|U_{\mu 3}|}{|U_{\tau 3}|}$ ratio of ν_μ and ν_τ component in ν_3

What we know – mixing

- approx. equal mixing of ν_μ and ν_τ in all mass states:
  $\theta_{23} \approx 45^\circ$ (with significant uncertainty)
- there is one mass state ("ν_1") which is dominantely ν_e ($\theta_{12} \approx 30^\circ$), and it is the lighter of the two states of the doublet with the small splitting (MSW in sun)
- there is a small ν_e component in the mass state ν_3: $\theta_{13} \approx 9^\circ$
  we do not know whether this mass state is the heaviest (normal ordering) or the lightest (inverted ordering)

Complementarity of global oscillation data

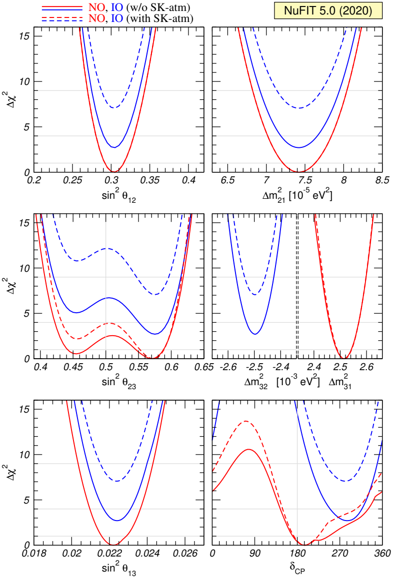
param	experiment	comment
θ_{12}	SNO, SuperK, (KamLAND)	resonant matter effect in the Sun
θ_{23}	SuperK, T2K, NOvA	ν_μ disappearance atmospheric (accelerator) neutrinos
θ_{13}	DayaBay, RENO, D-Chooz (T2K, NOvA)	$\bar{\nu}_e$ disappearance reactor experiments @ ~ 1 km
Δm^2_{21}	KamLAND, (SNO, SuperK)	$\bar{\nu}_e$ disappearance reactor @ ~ 180 km (spectrum)
$\lvert\Delta m^2_{31}\rvert$	MINOS, T2K, NOvA, DayaBay	ν_μ and $\bar{\nu}_e$ disapp (spectrum)
δ	T2K, NOvA + DayaBay	very weak sensitivity combination of $(\nu_\mu \to \nu_e) + \bar{\nu}_e$ disap

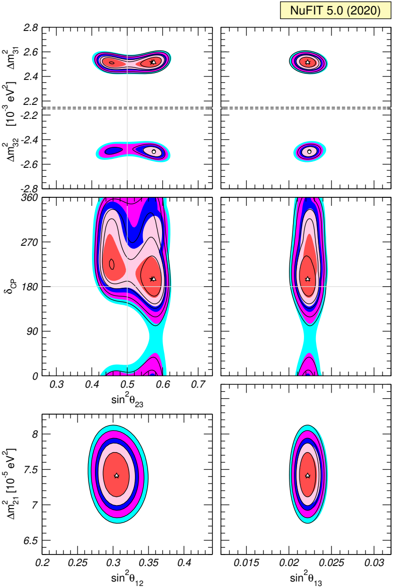
- global data fits nicely with the 3 neutrinos from the SM
- a few "anomalies" at 2-3 σ: LSND, MiniBooNE, reactor anomaly, no LMA MSW up-turn of solar neutrino spectrum – SOLVED 2020 (!)

Global 3-flavour fit

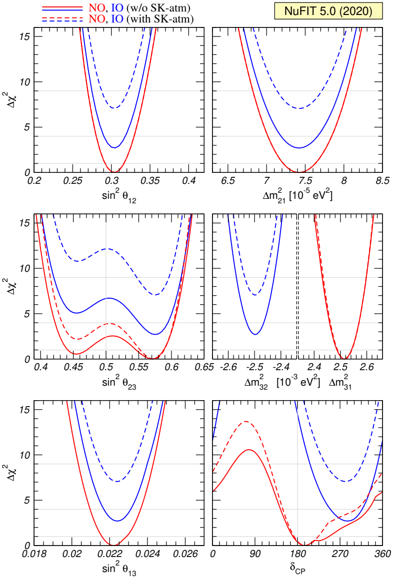
- NuFit collaboration: www.nu-fit.org
  with M.C. Gonzalez-Garcia, M. Maltoni, et al.
- latest paper:
  Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou, 2007.14792
- latest version: 5.0 (as of July 2020)
- provides updated global fit results
  tables & figures, χ^2 data for download

Global 3-flavour fit





Global 3-flavour fit



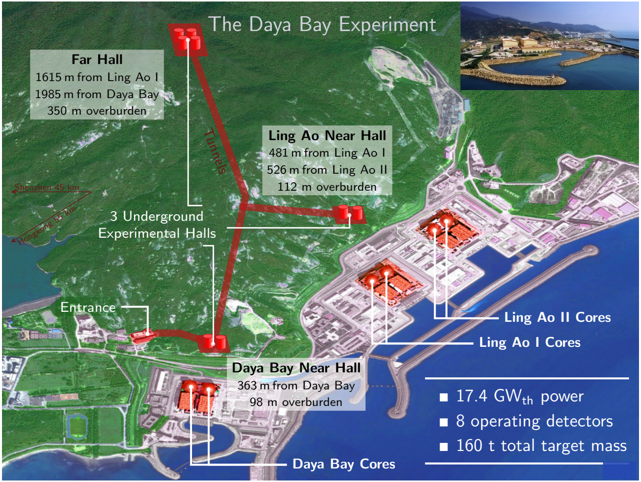
- robust determination (relat. precision at 3σ):

$$\begin{array}{ll} \theta_{12}\,(14\%) \quad , & \theta_{13}\,(9)\% \\ \Delta m^2_{21}\,(16\%)\,, & |\Delta m^2_{3\ell}|\,(6.7\%) \end{array}$$

- broad allowed range for $\theta_{23}\,(27\%)$, non-significant indications for non-maximality/octant
- ambiguity in sign of $\Delta m^2_{3\ell}$ → mass ordering
- values of $\delta_{\rm CP} \simeq 90^\circ$ disfavoured

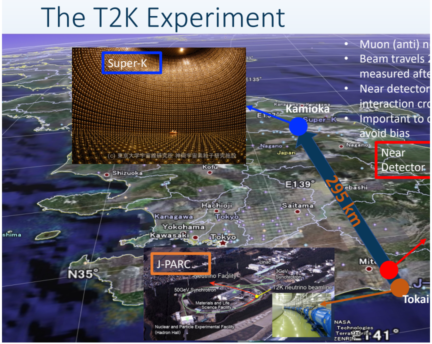
Daya Bay reactor experiment

- $\bar{\nu}_e \to \bar{\nu}_e$ disappearance



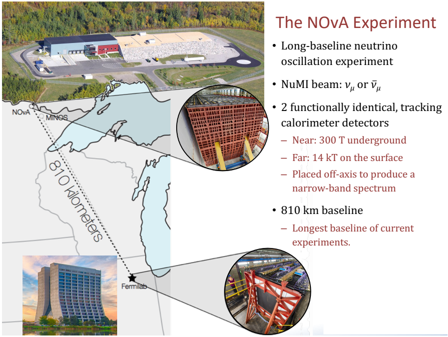
T2K and NOvA accelerator experiments

- $\nu_\mu \to \nu_\mu$ and $\bar{\nu}_\mu \to \bar{\nu}_\mu$ disappearance
- $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ appearance



The T2K Experiment

- Muon (anti) n
- Beam travels 2 measured afte
- Near detector interaction cro
- Important to c avoid bias



The NOvA Experiment

- Long-baseline neutrino oscillation experiment
- NuMI beam: ν_μ or $\bar{\nu}_\mu$
- 2 functionally identical, tracking calorimeter detectors
  - Near: 300 T underground
  - Far: 14 kT on the surface
  - Placed off-axis to produce a narrow-band spectrum
- 810 km baseline
  - Longest baseline of current experiments.

Disappearance due to Δm^2_{31}

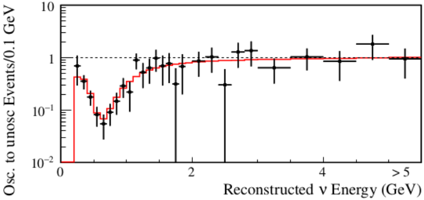
$$P_{\text{survival}} \approx 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4}\frac{L}{E_\nu}\right)$$

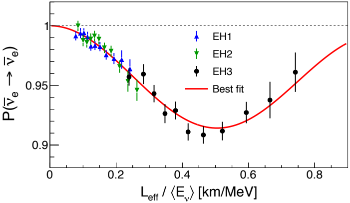
T2K, 2015

$\nu_\mu \to \nu_\mu$, $\langle L\rangle \sim 295$ km

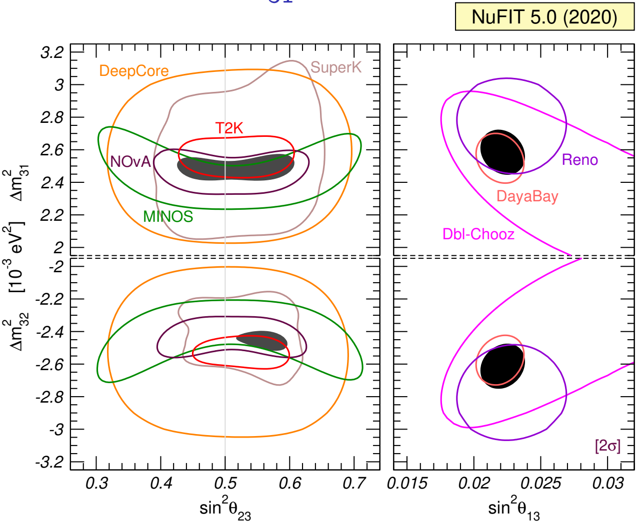
DayaBay, 2015

$\bar\nu_e \to \bar\nu_e$, $\langle L\rangle \sim 2$ km





Disappearance due to Δm^2_{31}



Complementarity between beam and reactor experiments

- $\nu_\mu \to \nu_e$ appearance probability (T2K, NOvA):

$$\boxed{\begin{aligned} P_{\mu e} \quad \approx \quad & \sin^2 2\theta_{13} \, \sin^2\theta_{23} \, \frac{\sin^2(1-A)\Delta}{(1-A)^2} \\ + \quad & \sin 2\theta_{13} \, \hat{\alpha} \, \sin 2\theta_{23} \, \frac{\sin(1-A)\Delta}{1-A} \frac{\sin A\Delta}{A} \, \cos(\Delta + \delta_{\mathrm{CP}}) \end{aligned}}$$

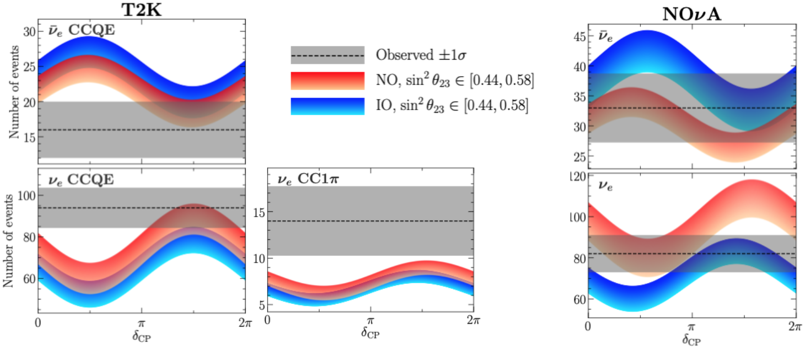
with $$\Delta \equiv \frac{\Delta m^2_{31} L}{4E_\nu} \,, \quad \hat{\alpha} \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \sin 2\theta_{12} \,, \quad A \equiv \frac{2E_\nu V}{\Delta m^2_{31}}$$

- ν_e survival probability (reactor experiments, e.g. Daya Bay)

$$\boxed{P_{ee} \approx 1 - \sin^2 2\theta_{13} \sin^2\Delta}$$

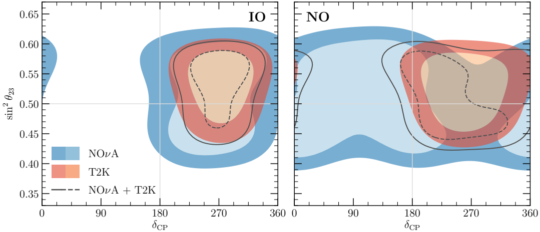
Latest restults from T2K and NOvA

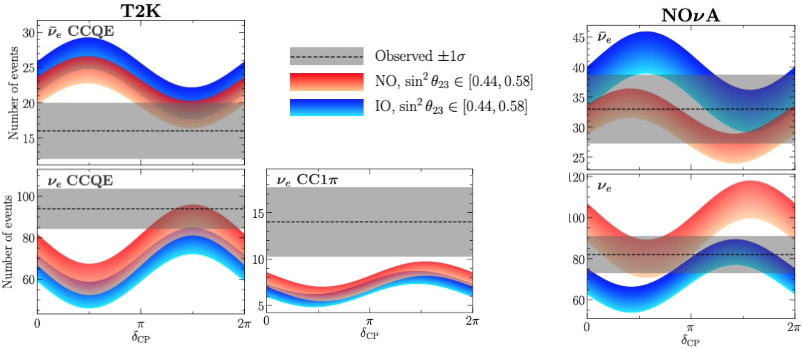
$$P_{\mu e} \approx \sin^2 2\theta_{13}\, \sin^2\theta_{23}\, \frac{\sin^2(1-A)\Delta}{(1-A)^2} + \sin 2\theta_{13}\, \hat{\alpha}\, \sin 2\theta_{23}\, \frac{\sin(1-A)\Delta}{1-A}\, \frac{\sin A\Delta}{A}\, \cos(\Delta + \delta_{\mathrm{CP}})$$



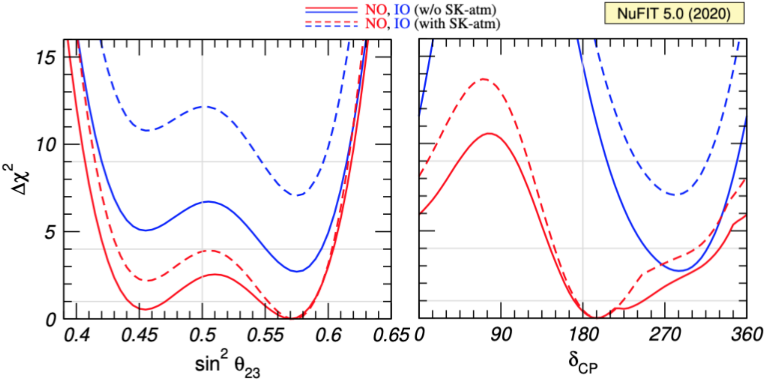
Latest restults from T2K and NOvA

Esteban et al., 2007.14792





Status of θ_{23} and δ_{CP}



Esteban et al., 2007.14792

CP violation in neutrino oscillations

Leptonic CP violation will manifest itself in a difference of the vacuum oscillation probabilities for neutrinos and anti-neutrinos
Cabibbo, 1977; Bilenky, Hosek, Petcov, 1980, Barger, Whisnant, Phillips, 1980

$$P_{\nu_\alpha\to\nu_\beta} - P_{\bar\nu_\alpha\to\bar\nu_\beta} = -16\, J_{\alpha\beta} \sin\frac{\Delta m^2_{21} L}{4E_\nu} \sin\frac{\Delta m^2_{32} L}{4E_\nu} \sin\frac{\Delta m^2_{31} L}{4E_\nu} \,,$$

where

$$J_{\alpha\beta} = \mathrm{Im}(U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2}) = \pm J \,,$$

with $+(-)$ for (anti-)cyclic permutation of the indices e, μ, τ.

J: leptonic analogue to the Jarlskog-invariant in the quark sector
Jarlskog, 1985

CP violation

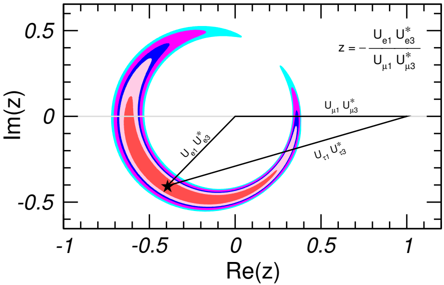
Jarlskog invariant:

$$J = |\mathrm{Im}(U_{\alpha 1} U^*_{\alpha 2} U^*_{\beta 1} U_{\beta 2})| = s_{12} c_{12} s_{23} c_{23} s_{13} c_{13}^2 \sin\delta \equiv J^{\max} \sin\delta$$

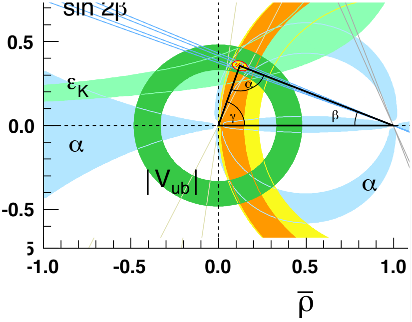
$$\boxed{J_{\mathrm{CP}}^{\max} = 0.0333 \pm 0.0006\,(\pm 0.0019) \text{ at } 1\sigma\ (3\sigma)}$$

$$J_{\mathrm{CP}}^{\mathrm{quarks}} = (3.18 \pm 0.15) \times 10^{-5}$$

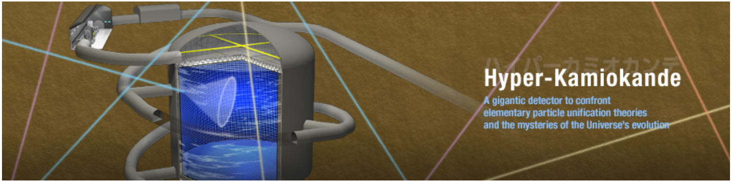
NuFIT 4.0 (2018)



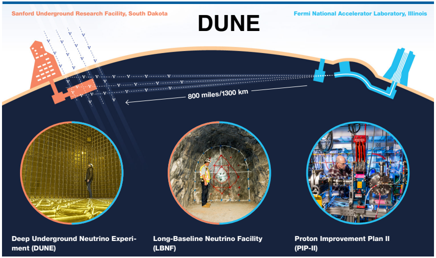
PDG 2018



T2K: J-PARC → HyperK (285 km, WC detector)



DUNE: Fermilab → Homestake (1300 km, LAr detectors)



oscillation science goals: determine mass ordering and CP phase

Determining the mass ordering

- Looking for the matter effect in transitions involving Δm^2_{31}
  - long-baseline accelerator experiments NOvA, DUNE
  - atmospheric neutrino experiments PINGU, ORCA, HyperK

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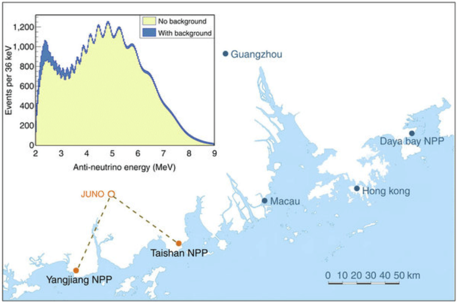
  - reactor experiment at 60 km JUNO



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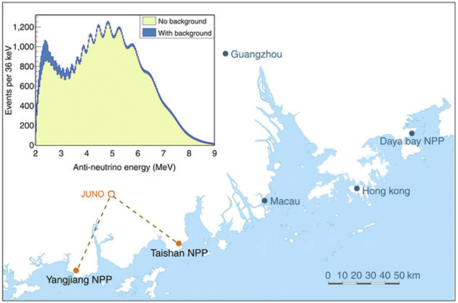

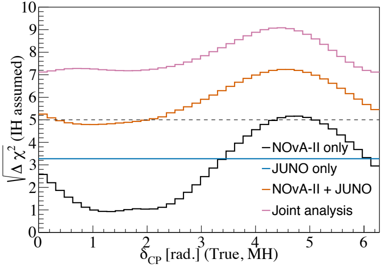

Cao et al., 2009.08585

Outline

Summary

- global data on neutrino oscillations is (mostly) consistent with 3-flavour oscillations
- at least two neutrinos are massive
- typical mass scales

$$\sqrt{\Delta m^2_{21}} \sim 0.0086\,\text{eV}$$
$$\sqrt{\Delta m^2_{31}} \sim 0.05\,\text{eV}$$

  are much smaller than all other fermion masses
- all three mixing angles are measured with reasonable precision
- lepton mixing is VERY different from quark mixing

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The SM flavour puzzle

Lepton mixing:

$$\theta_{12} \approx 33^\circ$$
$$\theta_{23} \approx 45^\circ$$
$$\theta_{13} \approx 9^\circ$$

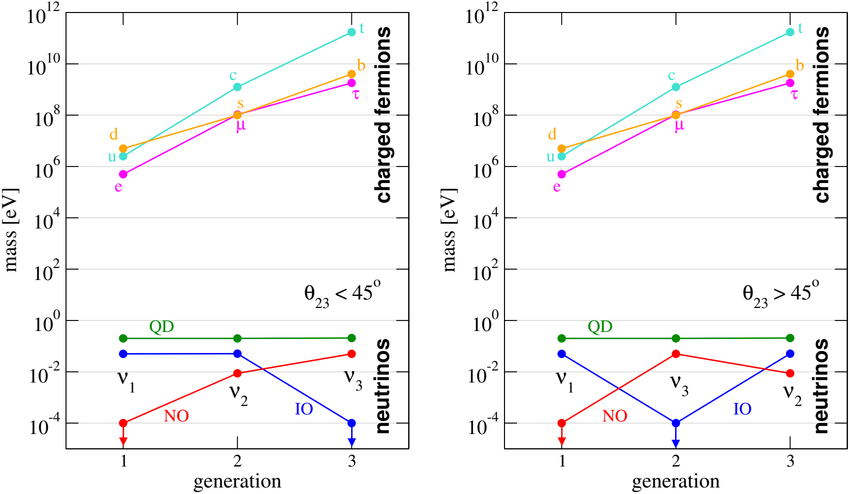
$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \epsilon \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}$$

Quark mixing:

$$\theta_{12} \approx 13^\circ$$
$$\theta_{23} \approx 2^\circ$$
$$\theta_{13} \approx 0.2^\circ$$

$$U_{CKM} = \begin{pmatrix} 1 & \epsilon & {\scriptscriptstyle \epsilon} \\ \epsilon & 1 & {\scriptscriptstyle \epsilon} \\ {\scriptscriptstyle \epsilon} & {\scriptscriptstyle \epsilon} & 1 \end{pmatrix}$$

The SM flavour puzzle



Summary

open questions for oscillation experiments:

- identify neutrino mass ordering
- establish leptonic CP violation
- precision measurments (e.g., $\theta_{23} \approx 45^\circ$?)
- over-constrain 3-flavour oscillations

questions which cannot be addressed by oscillations:

- absolute neutrino mass scale
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